ECONOMETRICS OF DISEQUILIBRIUM AND RATIONAL EXPECTATIONS

By

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This dissertation is dedicated to
the memory of my father,
the late Bibhuti Bhusan Chanda,
whose love has been my constant
companion.

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Since mid-60's, considerable interest has centered around the notion of disequilibrium as an alternative paradigm in economics. The interest has been equally shared by econometricians as well in estimation and inference of models of markets in disequilibrium.

In the early 70's, Lucas revolutionized a hypothesis about formation of expectations suggested by Muth about a decade ago. This expectation mechanism had been termed "rational" by Muth and was found to have major implications in macroeconomic modeling.

In this dissertation, an attempt has been made to provide a synthesis of two important approaches in economics by integrating "rational expectations" into the disequilibrium econometric literature. Attempts have also been made to resolve some of the unsatisfactory features of models of markets in disequilibrium currently used in the literature. This is done by introducing rational price expectations, spillovers, inventroeis, future price expectation, which are elements ignored in the current econometric literature.

An important class of disequilibrium models discussed in this dissertation are the models with controlled prices. The analysis is extended to multi-market disequilibrium models in a "partial equilibrium" framework that would have many interesting applications. Finally, an empirical examination is made of the corn market in the U.S. characterized by price-support program under alternative mechanisms of price expectation including the rational expectation hypothesis.

CHAPTER 1
INTRODUCTION

1.1 What is Disequilibrium?

The concept of equilibrium has been the central organizing idea in economics for a long time. It describes a state wherein the operating forces are so adjusted that there is no inherent tendency to change that state. For operational purposes, it refers to a situation in which prices clear the markets such that the traders are not required to recontract.

Disequilibrium, on the other hand, can be described in general terms as the absence or destruction of equilibrium. Interestingly enough, it encompasses so many different aspects of economic behavior that any economic analysis should take it into account to be meaningful. In fact, the concept of market equilibrium, i.e., the equality of supply and demand in markets, as used by Marshall, Walras and most subsequent writers in the neoclassical tradition is essentially based on the assumption that equilibrium occurs through a tâtonnement process in which the agents are allowed to recontract if the demand and supply fails to coincide at a price quoted by the mythical 'Walrasian auctioneer'. Since such markets in which agents are permitted to recontract are highly exceptional, the analysis of supply and demand under the assumption of equilibrium is not entirely satisfactory. As Professor Hicks remarked,

> in general, traders can not be expected to know just what total supplies are available on any market nor what total demands will be forthcoming at particular prices, any price fixed initially can only be a guess. It is not probable that demand and supply would actually be found to be equated at such a guessed price; if they are not, then in the course of trading, the price will move up and down. Now, if there is a change in price in the midst of trading, the situation appears to elude the ordinary apparatus of demand-and-supply analysis, for, strictly speaking, demand and supply curves give us amounts which buyers and sellers will demand and supply respectively at any price, if price is fixed at the start and adhered throughout. Earlier writers such as Walras and Edgeworth, had therefore supposed that the demand-and-supply analysis ought strictly to be confined to such markets as permitted of "recontract"...since such markets are highly exceptional, their solution of the problem (if it can be called one) was not very convincing (Hicks, 1946, pp. 127-128).

A meaningful analysis of supply and demand should therefore take into account the possibilities of disequilibrium as well. This is essentially what has been attempted in this thesis.

1.2 Sources of Disequilibrium

A fundamental question that lies at the core of any disequilibrium analysis is "what are the sources of disequilibrium?" Some of the analyses assume the prices to be exogenously determined and therefore the market does not necessarily clear. Disequilibrium may also arise in markets with exogenously fixed price limits. If the price hits the lower limit, we have excess supply and when the price hits the upper limit, we have excess demand (see, Maddala 1983a). The exogenous price rigidity or exogenously determined price units are, however, not necessary for a disequilibrium to occur. Grandmont (1977) asserted that in the real world, the agents have neither sufficient knowledge of the laws of the economic system nor sufficient computing abilities to forecast the future accurately. It is, therefore, only natural that in a real world, some adjustments would be made by quantity rationing if there is an excess demand or an excess supply in the market.

Bradfield and Zabel (1979), in a pioneering paper to examine how prices are determined in a real competitive market, studied the price formation in an organized security exchange using the specialist system to facilitate trading. The specialist is essentially a price-maker with exclusive knowledge of limit orders that arrive in the market randomly throughout the day. Market orders are revealed during the course of trading. He quotes two prices at periodic intervals, a 'bid' and an 'ask' price, and is obligated to buy a security at the bid and sell it at the ask price. Most trades take place at the bid and ask prices. Some brokers may, however, exchange among themselves within the bid and ask price (the spread).

The role of the specialist in such a market is essentially to maintain a 'fair and orderly market' in that security by trading on his own account to satisfy any excess demand in that market. Bradfield and Zabel appropriately pointed out that the specialist, like all other participants in the market, is interested in his own returns and quotes prices to maximize his gains. He is not obligated to balance the demand and supplies by the participants in the market. No one, therefore, explicitly attempts to drive the system in any direction. The important fact, therefore, is that 'a definition of equilibrium is irrelevant to a study of the trading process' (Bradfield and Zabel, 1979, p. 52). The analysis has been extended in subsequent papers to explore the properties of an optimal spread and in the analysis of inventories and price adjustment in a monopolistic trading process (Zabel, 1981a, 1981b).

The markets considered in this thesis are less organized than the securities market considered by Zabel. Further, we will be concentrating on the econometric problems connected with markets where disequilibrium arises from partial adjustment of prices or controls on the movement of prices.

1.3 Econometric Modeling of Disequilibrium

Disequilibrium econometric models have a long history. In fact all the "partial adjustment" models of the fifties and sixties have been named 'disequilibrium' models. These models assume the existence of two types of cost: (1) cost of being out of equilibrium and (2) cost of adjustment to the optimal value. Assuming the cost functions to be quadratic, cost minimization yields a partial adjustment model. The optimal values are first derived independent of costs of adjustment and then the adjustment costs are superimposed through the partial adjustment specification. (see e.g., Rosen and Nadiri, 1974; Bergstrom and Wymer, 1976; and Johnson and Taylor, 1977)

By contrast, in the recent econometric literature on disequilibrium which started with the paper by Fair and Jaffee (1972), the disequilibrium is expressed by the condition

$$Q_t = \mathrm{Min}(D_t, S_t) \qquad (1.3.1)$$

where D_t and S_t are the quantity demanded and supplied respectively and Q_t is the quantity transacted. The idea goes back to a paper by Frisch (1949). The model assumes that the short-side of the market prevails and that there is 'rationing'. If there is excess demand, the quantity supplied is 'rationed' to the demanders and if there is excess supply the suppliers are 'allocated' the demands. Thus the model is a rationing model.

The reason why $D_t \neq S_t$ is presumably that prices are fixed exogenously. Thus the model is also a 'fix-price' model. The 'fix-price' models, of course, make sense only in centrally planned economies. Fair and Jaffee also introduce a price adjustment equation into this model. Specifically, they postulate

$$\Delta P_t = \gamma(D_t - S_t) \tag{1.3.2}$$

where $\gamma > 0$ and $\Delta P_t = P_t - P_{t-1}$. There is, however, something peculiar with this price-adjutment equation. If P_{t-1} is the price fixed for period (t-1), then this equation says that the price for period t is adjusted up or down depending on whether $D_t > S_t$ or $D_t < S_t$. Note however that D_t and S_t are not observed before the end of the period. It is for this reason that Laffont and Garcia (1977) suggest an alternative price-adjustment equation

$$\Delta P_{t+1} = \gamma(D_t - S_t) \tag{1.3.3}$$

i.e., whoever is fixing prices changes the prices for the next period based on the current period's excess demand or excess supply. Even this price adjustment equation really begs the question as to why this price-fixing authority is fixing prices in such an aribtary fashion. Presumably, there should be a mechanism of forecasting D_t and S_t and the price fixed so that the forecast values of D_t and S_t are equal. A mechanism of this sort, called "anticipatory pricing" will be discussed in the next chapter.

There is one way of justifying equation (1.3.2). This is to assume that there exists an equilibrium price P_t^* that equates D_t and S_t. However, prices do not adjust to the equilibrium level because of institutional and other rigidities, i.e.,

$$P_t - P_{t-1} = \lambda(P_t^* - P_{t-1}) \qquad (1.3.4)$$

It is this sluggishness in the adjustment of prices that produces the disequilibrium. If we assume linear demand and supply functions, then equation (1.3.4) implies an equation of the type (1.3.2). This is the approach taken by Bowden (1978) and Maddala (1983b). The interpretation of equation (1.3.2) is not that prices are adjusted in response to excess demand or supply, as argued by Fair and Jaffee, but that $D_t \neq S_t$ because prices do not adjust to their market equilibrium level. As argued by Maddala (1983b), under this interpretation, it is equation (1.3.2) that is valid, not (1.3.3).

In this thesis we will be using the price adjustment equation (1.3.2) under the interpretation of partial adjustment of prices to their market equilibrating levels.

1.4 Expectations in Econometric Disequilibrium Models

One major element missing in the recent econometric disequilibrium models discussed in the previous section is that of price expectations. It is reasonable to assume that supply decisions (and possibly demand decisions as well) depend not on current prices but expected prices. Our objective in this thesis is to introduce price expectations into the disequilibrium econometric models and illustrate their use.

The introduction of price expectations in the model necessitates the formulation of a mechanism of formation of expectations. We will assume throughout our analysis that expectations are formed "rationally" in the sense that expectations are the same as the predictions of the model contingent on the information set the economic agents are assumed to have, i.e.,

$$P_t^e = E\ (P_t | I_{t-1}) \qquad (1.4.1)$$

where I_{t-1} represents the information set available to the economic agents at time t-1. (see Muth, 1961)

The estimation of a model in disequilibrium considered earlier is much more difficult than estimating an equilibrium model. It is obvious that the problem will be far more complicated with the introduction of price-expectations in the model, as the data on expected price are normally not available. Furthermore, if the model requires future expectations, there is a problem of multiplicity of solutions. In spite of the complexities involved, an integration of these two major ideas in economics is likely to have important policy implications and may, in many cases, turn out to be a more appropriate description of reality. This point will be investigated later with specific reference to the U.S. corn market.

One important point that needs to be clarified is the way that equation (1.4.1) is used in deriving the estimable equations in the models considered. That economic agents should use all the information availabe at the time they are forming expectations about the future is incontrovertible. The question is how to derive an expression for P_t^e based on this postulate. The procedure that is almost universally followed in all the econometric work involving "rational" expectations is to derive a reduced form equation for P_t from the model considered, and then take the expectation of P_t from this equation conditional on the information at time t-1. Obviously, the expression for P_t^e that we obtain by this procedure will be different for different models. Instead of using the word "rational" to describe the expectations so derived, we might as well use the term "model consistent". Thus, throughout the dissertation the term "rational" should be interpreted as "model consistent" and the latter terminology is not used simply because the term "rational" is the accepted terminology. It is clear that the important assumption implicit in all this analysis is that economic agents forming their expectations use exactly the same model as the one the econometrician is using. This is hardly a justifiable assumption. A more justifiable expression for P_t^e would be obtained by using the equation (1.4.1) as it is, without any specific reference to any particular model that the econometrician might be estimating. This would imply that we regress P_t on all the variables included in the information set I_{t-1} and take the predicted value of P_t from this equation as the value of P^e_t. The only problem with this procedure is that the information set I_{t-1} can include a very large number of variables. In the usual econometric work this problem is essentially 'solved' by confining our attention to the reduced form for P_t from the assumed model and then taking its conditional expectation, conditional on the information set I_{t-1}. This economizes on the number of explanatory variables and also produces cross-equation restrictions (see Wallis, 1980). It is also often argued that a test for such cross-equation restrictions is a test for the "rational expectations hypothesis" but as is clear such tests are merely tests of the hypothesis that economic agents forming expectations are using precisely the model that the econometrician is estimating.

Because of all these problems, in the empirical work reported in Chapters 4 and 5 we will be using alternative expectations mechanisms and comparing them with the so-called "rational" expectations hypothesis which, as argued above, is merely a "model-consistent" method of forming expectations. However, we will use the word "rational" throughout the thesis since that is the accepted terminology. Each of these alternative expectations mechanisms can also be termed 'rational' for some definition of the information set I_{t-1}.

1.5 An Outline of the Thesis

The preceding sections give a brief introduction to the econometric disequilibrium models and the need to incorporate expectations into these models.

In Chapter 2 we discuss in detail the econometric disequilibrium model under partial adjustment of prices. We introduce rational (model consistent) price expectations, spillovers, and inventories, which are elements ignored in the existing econometric literature. We also discuss anticipatory pricing and serial correlation problems in the context of this model. Tests for disequilibrium in this model are extended to the case where expectations of prices are introduced.

In Chapter 3 we discuss another set of disequilibrium models--the controlled price models. We discuss how expectations are formed under a system of controlled prices. We also extend the analysis to multimarket problems with spillovers, where the markets for substitute goods are in equilibrium.

An important application of disequilibrium models is in markets with controlled prices. Markets for feedgrain in the U.S. have been characterized by price supports for a long time. Moreover, a biological time lag separating the production decision and the final output makes it incumbent on the producers in the agricultural markets to make their production decisions on the basis of an expected price that will prevail in the market when the crop is harvested. In Chapter 4, therefore, we examine an econometric model of the corn market in the U.S. under price support program with price expectations on the supply side of the market. Corn is, by far, the major feedgrain in the U.S. accounting for nearly two-thirds of the total production of feedgrains in the country and serves as an important link between the crop and the livestock sector. Any analysis that explicitly accounts for the effects of price support programs on supply and demand should, therefore, be of considerable interest and possess important policy implications.

In Chapter 5, the empirical results for the U.S. corn market under price support are presented for alternative mechanisms of expectation formation. The data pertains to 1949-1981 period over which the price support program has remained effective in the corn market in the U.S. The findings are indicative of how price expectations are formed in such markets and how price support programs influence the supply and demand of corn in the U.S.

Finally, Chpater 6 presents the conclusions of this study.

CHPATER 2
ESTIMATION OF MODELS OF DISEQUILIBRIUM UNDER PARTIAL ADJUSTMENT OF PRICES

2.1 Introduction

In the preceding chapter we gave a brief outline of the econometric disequilibrium models. The main characteristic is that $D_t \neq S_t$, which can arise from several sources:

(i) Prices are exogenously fixed: Such models are called 'fix-price' models. In this case there is no role for expectations of prices. We will not deal with this case since these models are relevant only in the case of controlled economies.

(ii) Prices do not adjust to the market equilibrating levels: These are 'partial adjustment' models. There is scope for price expectations in these models. This is the class of models discussed in this chapter.

(iii) Prices are controlled to lie within some limits. Markets with regulated prices (rent control, wage control, price control, price support, etc.), fall in this category. Again, there is a scope for price expectations in these models. These models are discussed in the next chapter.

2.2 A Simple Disequilibrium Model Under Rational Expectations

Our model consists of

Demand function: $D_t = \beta_1' X_{1t} + \alpha_1 P_t + U_{1t}$ (2.2.1)

Supply function: $S_t = \beta_2' X_{2t} + \alpha_2 P_t^e + U_{2t}$ (2.2.2)

Minimum function: $Q_t = \text{Min}\ (D_t, S_t)$ (2.2.3)

Price-adjustment equation: $\Delta P_t = \gamma(D_t - S_t),\ \gamma > 0$ (2.2.4)

where X_{1t}, X_{2t} are vectors of exogenous variables; U_{1t}, U_{2t} are error terms (usually assumed to be jointly normal, covariance matrix Σ) serially uncorrelated and where D_t and S_t are unobserved by the econometrician but Q_t is observed.

Under the rational expectations hypothesis, we also have the following equation,

$$P_t^e = E(P_t | I_{t-1}) \tag{2.2.5}$$

where P_t^e is the expected price and I_{t-1} represents the information set the economic agents are assumed to have.

Equation (2.2.5) implies that we can write

$$P_t = P_t^e + V_t$$

where V_t is uncorrelated with all the variables in the information set I_{t-1}. If the information set I_{t-1} inlcudes the exogenous variables X_{1t} and X_{2t}, i.e., if these exogenous variables are known at time t-1, then we can substitute $P_t^e = P_t - V_t$ in equation (2.2.2). We can re-define a residual $U_{2t}^* = U_{2t} - \alpha_2 V_t$ and U_{2t}^* has the same properties as U_{2t}.

Thus the estimation of the model simplifies to the case considered by Fair and Jaffee (1972). The estimation of this model is discussed in Amemiya (1974).

If, on the other hand, X_{1t} and X_{2t} are now known at time (t-1) we cannot treat V_t the same way as we treat U_{2t} since V_t can be correlated with X_{1t} and X_{2t}. In this case we proceed as follows.

From equations (2.2.1), (2.2.2), and (2.2.4), we have

$$\Delta P_t = \gamma(D_t - S_t)$$

$$P_t - P_{t-1} = \gamma[(\beta_1' X_{1t} - \beta_2' X_{2t}) + (\alpha_1 P_t - \alpha_2 P_t^e) + (U_{1t} - U_{2t})]$$

Taking expectations of both sides conditional on the information set I_{t-1},

$$P_t^e - P_{t-1} = \gamma[\beta_1' X_{1t}^* - \beta_2' X_{2t}^*) + (\alpha_1 - \alpha_2) P_t^e]$$

$$P_t^e = \lambda_1(\beta_1' X_{1t}^* - \beta_2' X_{2t}^*) + \lambda_2 P_{t-1} \qquad (2.2.6)$$

where

$$\lambda_1 = \frac{\gamma}{[1 + \gamma(\alpha_2 - \alpha_1)]}$$

$$\lambda_2 = \frac{1}{[1 + \gamma(\alpha_2 - \alpha_1)]}$$

and X_{1t}^* and X_{2t}^* are the expected values of X_{1t} and X_{2t}.

To obtain X_{1t}^* and X_{2t}^* we have to make some assumptions about how these exogenous variables are generated. A common assumption is that they follow vector autoregressive processes. Let us for the sake of simplicity of notation assume a first order autoregressive process.

$$X_{1t} = \phi_{11} X_{1,t-1} + \phi_{12} X_{2,t-1} + \varepsilon_{1t}$$

$$X_{2t} = \phi_{21} X_{1,t-1} + \phi_{22} X_{2,t-1} + \varepsilon_{2t} \qquad (2.2.7)$$

Then

$$X^*_{1t} = \phi_{11} X_{1,t-1} + \phi_{12} X_{2,t-1}$$

and

$$X^*_{2t} = \phi_{21} X_{1,t-1} + \phi_{22} X_{2,t-1} \qquad (2.2.8)$$

We substitute these equations in (2.2.6) and substitute the resulting expression for P^e_t in equation (2.2.2).

The estimation of the model will proceed as with the usual disequilibrium model. The likelihood function in this model is derived in exactly the same way as with the Fair and Jaffee model, as derived in Amemiya (1974). The only extra complication is the existence of cross-equation restrictions as implied by equations (2.2.7), as discussed in Wallis (1980).

2.3 Two-Stage Least Squares Estimation

The two-stage least squares estimation suggested in Amemiya (1974) can be easily adapted to the above model. We discuss this for two reasons. Firstly, it enables us to obtain some consistent estimates of the parameters for use in the estimation of maximum likelihood. Secondly, we will need these expressions in the discussion of serial correlation problems in the next section.

Our model consists of equations (2.2.1) through (2.2.5). In view of (2.2.4), for $\Delta P_t > 0$ we have

$$D_t > S_t$$

Hence $S_t = Q_t$ and we have

$$Q_t = \beta_2' X_{2t} + \alpha_2 P_t^e + U_{2t} \qquad (2.3.1)$$

where we assume that we substitute the expression for P_t^e as obtained in (2.2.6) and (2.2.8).

We may also write (2.2.4) as

$$Q_t = D_t - \frac{1}{\gamma} \Delta P_t$$

So, we have

$$Q_t = \beta_1' X_{1t} + \alpha_1 P_t + U_{1t} - \frac{1}{\gamma} \Delta P_t \qquad (2.3.2)$$

Similarly, for $\Delta P_t < 0$, we have

$$D_t < S_t$$

and hence

$$D_t = Q_t$$

Therefore,

$$Q_t = \beta_1' X_{1t} + \alpha_1 P_t + U_{1t} \qquad (2.3.3)$$

Also, rewriting (2.2.4) as $Q_t = S_t + \frac{1}{\gamma} \Delta P_t$, we have

$$Q_t = \beta_2' X_{2t} + \alpha_2 P_t^e + U_{2t} + \frac{1}{\gamma} \Delta P_t \qquad (2.3.4)$$

Again, we assume that we substitute the expression for P_t^e as obtained in (2.2.6) and (2.2.8). Combining equations (2.3.2) and (2.3.3), we get

$$Q_t = \beta_1' X_{1t} + \alpha_1 P_t + \frac{1}{\gamma} Z_{1t} + U_{1t} \tag{2.3.5}$$

$$\begin{aligned} Z_{1t} &= -\Delta P_t \text{ if } \Delta P_t > 0 \\ &= 0 \quad \text{ if } \Delta P_t < 0 \end{aligned}$$

Similarly, combining equations (2.3.1) and (2.3.4), we get

$$Q_t = \beta_2' X_{2t} + \alpha_2 P_t^e + \frac{1}{\gamma} Z_{2t} + U_{2t} \tag{2.3.6}$$

where

$$\begin{aligned} Z_{2t} &= 0 \quad \text{ if } \Delta P_t > 0 \\ &= \Delta P_t \text{ if } \Delta P_t < 0 \end{aligned}$$

and P_t^e is given by (2.2.6) and (2.2.8).

The only modification in the procedure outlined in Amemiya (1974) that we need to make is that equations (2.3.5) and (2.3.6) have to be estimated jointly with equations (2.2.7), taking into account the cross-equation restrictions.

2.4 Serially Correlated Errors

Most of the econometric analyses of disequilibrium models have been done under the assumption that the residuals U_{1t} and U_{2t} are not serially correlated. This is a serious assumption and there seems to be very little that could be done about it because when the residuals are serially correlated, the likelihood function becomes very complicated and the maximum likelihood method of estimation becomes almost intractable. In this section, we will outline a method of estimation of our model when the residuals are assumed to be serially correlated.

Let us assume that U_{it} $(i = 1,2)$ follows a first-order Markov provess, viz.

$$U_{it} = \rho_i U_{it-1} + \varepsilon_{it} \tag{2.4.1}$$

where $\varepsilon_{it} \sim N(0,\sigma_i^2)$ and $|\rho_i| < 1$.

From equations (2.3.5) and (2.4.1) we have

$$\begin{aligned} Q_t = \rho_1 Q_{t-1} &+ \beta_1'(X_{1t} - \rho_1 X_{1t-1}) + \alpha_1'(P_t - \rho_1 P_{t-1}) \\ &+ \frac{1}{\gamma}(Z_{1t} - \rho_1 Z_{1t-1}) + \varepsilon_{1t} \end{aligned} \tag{2.4.2}$$

Similarly, from (2.3.6) and (2.4.1), we may write

$$\begin{aligned} Q_t = \rho_2 Q_{t-1} &+ \beta_2'(X_{2t} - \rho_2 Z_{2t-1}) + \alpha_2(P_t^e - \rho_2 P_{t-1}^e) \\ &+ \frac{1}{\gamma}(Z_{2t} - \rho_2 Z_{2t-1}) + \varepsilon_{2t} \end{aligned} \tag{2.4.3}$$

We can now substitute the expression for P_t^e from equations (2.2.6) and (2.2.8) and estimate equations (2.4.2) and (2.4.3) along with equations (2.2.7) by non-linear least squares procedures taking account of the cross-equation restrictions. Though cumbersome, that is at least a feasible procedure as compared to the method of maximum likelihood that depends on integrals of a high dimension.

2.5 The Model with Anticipatory Pricing

We have argued that the price adjustment equation (2.2.4) has to be interpreted as an equation arising from partial adjustment of prices to their market equilibrating level. We have also said that if we are in a fix-price framework, there is no reason why anyone should fix prices following a rule given by equation (2.2.4). There is, however, one form of a fix-price model that does make sense and that is somewhat related to the models with rational expectations that we have been considering. This fix-price rule is to fix prices at a level that equates the expected demand and supply during the period under consideration. It has been called an "anticipatory pricing rule" by Green and Laffont (1981).

Consider a simple disequilibrium model

$$D_t = \beta_1' X_{1t} + \alpha_1 P_t + U_{1t} \tag{2.5.1}$$

$$S_t = \beta_2' X_{2t} + \alpha_2 P_t + U_{2t} \tag{2.5.2}$$

$$Q_t = \text{Min}\ (D_t, S_t)$$

where

D_t is the quantity demanded

S_t is the quantity supplied

Q_t is the quantity transacted

P_t denotes the price

X_{1t}, X_{2t} are sets of exogenous variables and U_{1t}, U_{2t} are error terms distributed jointly normal with mean vector zero and dispersion matrix Σ, and serially uncorrelated over time.

Under anticipatory pricing mechanism, the prices will be determined at the beginning of the period at the market clearing level if there were to be no unforeseen disturbances in the demand and supply functions. This P_t, under the anticipatory pricing mechanism would be determined as

$$P_t = \frac{\beta_2' X_{2t}^* - \beta_1' X_{1t}^*}{\alpha_1 - \alpha_2} \tag{2.5.4}$$

where X_{2t}^* and X_{1t}^* are the predicted values of X_{2t} and X_{1t} respectively.

If we make the assumption that X_{1t} and X_{2t} are generated by the process (2.2.7) then we substitute the values for X_{1t}^* and X_{2t}^* as given by equations (2.2.8) into equation (2.5.4) and substitute the resulting expression for P_t in equations (2.5.1) and (2.5.2). The estimation of the model now is the same as that outlined in Maddala and Nelson (1974) for the model with unknown sample separation except for the fact that there are non-linear constraints across equations.

The purpose of the above discussion was to outline how a meaningful fix-price model is similar in structure to the models with rational expectations we have been considering. The major differences is that in this case we end up with a switching model with unknown sample separation whereas with the models with partial adjustment in prices we have a model with known sample separation. Models with unknown sample separation pose problems of badly behaved likelihood functions and hence we will concentrate on the partial adjustment models rather than the fix-price models.

2.6 A Model with Future Expectations

Our models, considered so far, have included the expectations of the economic agents regarding the price that will prevail in the market at time t (based on the information available at time t-1) to determine the quantity to be supplied at time t. In this section, we will extend the analysis by incorporating future price-expectations in the model. In macroeconomic modelling under rational expectations, inclusion of future price-expectations has been an area of considerable interest (see e.g., Blanchard, 1979; Blanchard and Kahn, 1980; etc.). We will do so by assuming that the demand function is not only dependent on the current price level but also on the price that the consumers expect to prevail at time t+1.

Such information has an obvious intuitive appeal. If the price of the automobile is expected to go up (down) next year relative to the current price, it would certainly influence the current demand for automobiles. In many economies, in fact, shortage has been known to occur since the consumers expected that a shortage might occur in the next period and therefore, they scaled up their purchase to avoid any future hardship and thus bringing about a shortage in reality. Although we do not discuss any explicit source of disequilibrium in this analysis, the experience of shortage economies should provide additional justification for inclusion of future price expectations in the demand function in modelling markets in disequilibrium.

Our model consists of the following equations (for simplicity of exposition, we ignore the exogenous variables):

Demand function: $D_t = \alpha_1 P_t + \alpha_2({}_tE(P_{t+1}) - P_t) + U_{1t}$ (2.6.1)

Supply function: $S_t = \alpha_3 \, {}_{t-1}E(P_t) + U_{2t}$ (2.6.2)

Minimum condition: $Q_t = \text{Min } (D_t, S_t)$ (2.6.3)

Price-adjustment equation: $\Delta P_t = \gamma(D_t - S_t)$ (2.6.4)

where $\Delta P_t = P_t - P_{t-1}$

and ${}_{t-1}E(P_t) = E(P_t | I_{t-1})$

where I_{t-1} is the information available to the economic agents at time t-1. U_{1t}, U_{2t} are error terms, usually assumed to be jointly normal with mean vector zero, covariance matrix Σ and serially uncorrelated. Further, we assume

$$\alpha_1 < 0, \ \alpha_2 > 0, \ \alpha_3 > 0, \ \gamma > 0$$

The assumption $\alpha_1 < 0$ essentially implies that the market is more responsive to the current price than to the future expectations which is economically meaningful since the agents can be reasonably assumed to discount the future.

To obtain the reduced form for ${}_{t-1}E(P_t)$, we employ Lucas' method of undetermined coefficients in this case for which we proceed as follows:

Combining equations (2.6.1), (2.6.2), and (2.6.4), we have

$$P_t - P_{t-1} = \gamma[(\alpha_1 - \alpha_2)P_t + \alpha_2 \, {}_tE(P_{t+1}) - \alpha_3 \, {}_{t-1}E(P_t) + \varepsilon_t] \qquad (2.6.5)$$

where

$$\varepsilon_t = U_{1t} - U_{2t}$$

Assume the following representation of the reduced form for P_t, viz.

$$P_t = \pi_1 P_{t-1} + \pi_2 \varepsilon_t \qquad (2.6.6)$$

where π_1 and π_2 are the coefficients to be determined that characterize the distribution of P_t.

Taking expectations of both sides of (2.6.6) conditional on I_{t-1}, we have

$$_{t-1}E(P_t) = \pi_1 P_{t-1} \qquad (2.6.7)$$

Again, updating t by t+1 in equation (2.6.6) and taking expectations of both sides conditional on I_{t-1}, we get

$$_tE(P_{t+1}) = \pi_1 P_t \qquad (2.6.8)$$

From (2.6.5), (2.6.7), and (2.6.8), we have

$$P_t - P_{t-1} = \gamma[(\alpha_1 - \alpha_2)P_t + \alpha_2\pi_1\ P_t - \alpha_3\ {}_{t-1}E(P_t) + \varepsilon_t]$$

which on simplification yields

$$(1 - \gamma(\alpha_1 - \alpha_2 + \pi_1\alpha_2))P_t = (1 - \gamma\alpha_3\pi_1)P_{t-1} + \gamma\varepsilon_t \qquad (2.6.9)$$

Now, in view of (2.6.6), equation (2.6.9) can be rewritten as

$$(1 - \gamma(\alpha_1 - \alpha_2 + \pi_1\alpha_2))(\pi_1 P_{t-1} + \pi_2\varepsilon_t)$$

$$= (1 - \gamma\alpha_3\pi_1)P_{t-1} + \gamma\varepsilon_t \qquad (2.6.10)$$

Equating the coefficients of ε_t on both sides, we get

$$(1 - \gamma(\alpha_1 - \alpha_2 + \pi_1\alpha_2))\pi_2 = \gamma$$

Therefore,

$$\pi_2 = \frac{\gamma}{1 - \gamma(\alpha_1 - \alpha_2 + \pi_1\alpha_2)} \qquad (2.6.11)$$

Again, equating coefficients of P_{t-1} on both sides of (2.6.10), we have

$$(1 - \gamma(\alpha_1 - \alpha_2 + \pi_1\alpha_2))\pi_1 = 1 - \gamma\alpha_3\pi_1$$

i.e.,

$$\gamma\alpha_2\pi_1^2 - (1 - \gamma(\alpha_1 - \alpha_2 - \alpha_3))\pi_1 + 1 = 0 \qquad (2.6.12)$$

The solution for π_1 is, therefore, given by the roots of the quadratic equation (2.6.12), viz.

$$\pi_1 = \frac{1 - \gamma(\alpha_1 - \alpha_2 - \alpha_3) \pm \sqrt{(1 - \gamma(\alpha_1 - \alpha_2 - \alpha_3))^2 - 4\gamma\alpha_2}}{2\gamma\alpha_2} \qquad (2.6.13)$$

In view of (2.6.13), the condition for existence of a reduced form for P_t is given by

$$(1 - \gamma(\alpha_1 - \alpha_2 - \alpha_3))^2 - 4\gamma\alpha_2 > 0 \qquad (2.6.14)$$

Note that when the condition for existence is satisfied, we have two solutions for π_1. Thus there will be multiple solutions for P_t in this case. This is usually the case in the rational expectations literature whenever future expectations appear in the model (see e.g., Taylor, 1977; Blanchard, 1979; Blanchard and Kahn, 1980; etc.). This essentially means that there are two price-distributions for this model satisfying the condition of rationality and therefore, to obtain a unique condition, further conditions must be imposed.

A criterion that has often been successfully utilized to ensure uniqueness is the stability of the conditionally expected price path generated by the distribution. In stochastic models where the unconditional mean price is well determined, this is equivalent to requiring finiteness of the variance of the price distribution. It should however be noted that this criterion, by itself does not necessarily guarantee uniqueness as there may be many price distributions with finite variance satisfying the requirements of rational expectations (see e.g., Taylor, 1977). In such cases, further restrictions such as minimum variance, etc., ought to be imposed to ensure uniqueness of the price distribution.

For a price distribution determined by (2.6.6) to be stable, we require

$$|\pi_1| < 1$$

Or in other words,

$$-1 < \frac{1-\gamma(\alpha_1-\alpha_2-\alpha_3) \pm \sqrt{(1-\gamma(\alpha_1-\alpha_1-\alpha_3))^2 - 4\gamma\alpha_2}}{2\gamma\alpha_2} < 1 \tag{2.6.15}$$

For notational simplicity, write $\beta = 1 - \gamma(\alpha_1 - \alpha_2 - \alpha_3)$. Then, (2.6.15) is equivalent to

$$-(2\gamma\alpha_2 + \beta) < \pm \sqrt{\beta^2 - 4\gamma\alpha_2} < (2\gamma\alpha_2 - \beta) \qquad (2.6.16)$$

Let us now examine if the condition for stability as warranted by (2.6.16) is satisfied by the price distribution corresponding to the smaller root (i.e., the one with negative sign before the discriminant in (2.6.13)).

Observe that

$$-(2\gamma\alpha_2 + \beta) < - \sqrt{\beta^2 - 4\gamma\alpha_2}$$

if $$(2\gamma\alpha_2 + \beta)^2 > \beta^2 - 4\gamma\alpha_2$$

i.e., if $$4\gamma^2\alpha_2^2 + \beta^2 + 4\beta\gamma\alpha_2 > \beta^2 - 4\gamma\alpha_2$$

i.e., if $$\gamma\alpha_2 + \beta + 1 > 0$$

which is true since β is positive (> 1).

Now, if $2\gamma\alpha_2 - \beta > 0$, the condition (2.6.16) is trivially satisfied by the smaller root. If, however, $2\gamma\alpha_2 - \beta < 0$, (2.6.16) would be satisfied if

$$- \sqrt{\beta^2-4\gamma\alpha_2} < (2\gamma\alpha_2-\beta)$$

provided $$\beta^2-4\gamma\alpha_2 > 4\gamma^2\alpha_2^2 + \beta^2-4\gamma\alpha_2$$

i.e., if $$\gamma\alpha_2 - \beta + 1 < 0$$

i.e., if $$\gamma(\alpha_1-\alpha_3) < 0$$ which is true.

Therefore, the condition for stability as required under (2.6.16) is satisfied by the smaller root corresponding to the price distributions determined by (2.6.6).

Further, it is easy to observe that price distribution corresponding to the larger root (as determined by the positive sign before the discriminant in (2.6.13) does not satisfy the condition of finite variance. This is because stability requires

$$\sqrt{\beta^2-4\gamma\alpha_2} < 2\gamma\alpha_2 - \beta \tag{2.6.17}$$

Now, if $2\gamma\alpha_2 - \beta < 0$, (2.6.17) is trivially not true. If however, $2\gamma\alpha_2 - \beta > 0$, then

$$\sqrt{\beta^2-2\gamma\alpha_2} < 2\gamma\alpha_2 - \beta$$

provided $\beta^2-4\gamma\alpha_2 < 4\gamma^2\alpha_2^2 + \beta^2 - 4\alpha\gamma_2\beta$

i.e., if $\gamma(\alpha_1-\alpha_3) > 0$ which is not true,

and therefore, (2.6.17) can not hold.

Therefore, the unique price distribution as determined by the criterion of finite variance corresponds to the smaller root in (2.6.13) above and is generated by (2.6.6).

So, in view of (2.6.7) and (2.6.8), we may now express the demand and the supply functions in terms of observables and parameters and proceed with estimation of the model with the technique outlined in the previous sections.

2.7 Models with Spillovers

We will now extend the analysis to models with spillovers of current excess demand and excess supply to subsequent periods. Such models have been considered by Laffont and Monfort (1979) but with no price expectations.

Let us consider a simple disequilibrium model where dynamic behavior is explicitly characterized through quantity adjustments, i.e., the unsatisfied demand or supply of the previous period enter as explanatory variables for the demand and supply specifications.

$$D_t = \beta_1' X_{1t} + \alpha_1 P_t + \alpha_2 (D_{t-1} - Q_{t-1}) + U_{1t} \tag{2.7.1}$$

$$S_t = \beta_2' X_{2t} + \lambda_1 P_t^e + \lambda_2 (S_{t-1} - Q_{t-1}) + U_{2t} \tag{2.7.2}$$

$$Q_t = \text{Min } (D_t, S_t) \tag{2.7.3}$$

and

$$\Delta P_t = \gamma (D_t - S_t) \tag{2.7.4}$$

Where U_{1t} and U_{2t} are assumed to be serially uncorrelated over time with zero mean and variances σ_1^2 and σ_2^2, respectively.

From equation (2.7.4), for $\Delta P_{t-1} > 0$, $D_{t-1} > S_{t-1}$. Hence $S_{t-1} = Q_{t-1}$

$$D_{t-1} - Q_{t-1} = D_{t-1} - S_{t-1} = \frac{1}{\gamma} \Delta P_{t-1}$$

Therefore, for $\Delta P_{t-1} > 0$, we have

$$D_t = \beta_1' X_{1t} + \alpha_1 P_t + \frac{\alpha_2}{\gamma} \Delta P_{t-1} + U_{1t} \tag{2.7.5}$$

$$S_t = \beta_2' X_{2t} + \gamma_1 P_t^e + U_{2t} \tag{2.7.6}$$

Similarly, for $\Delta P_{t-1} < 0$

$$D_t = \beta_1' X_{1t} + \alpha_1 P_t + U_{1t} \qquad (2.7.7)$$

$$S_t = \beta_2' X_{2t} + \lambda_1 P_t^e - \frac{\alpha_2}{\gamma} \Delta P_{t-1} + U_{2t} \qquad (2.7.8)$$

Now combining equations (2.7.5) and (2.7.7), we may rewrite the demand equation as

$$D_t = \beta_1' X_{1t} + \alpha_1 P_t + \frac{\alpha_2}{\gamma} Z_{1t-1} + U_{1t} \qquad (2.7.9)$$

where

$$\begin{array}{lll} Z_{1t} = \Delta P_t & \text{if} & \Delta P_t > 0 \\ \quad 0 & & \text{otherwise} \end{array}$$

Similarly, combining equations (2.7.6) and (2.7.8) we have

$$S_t = \beta_2' X_{2t} + \lambda_1 P_t^e + \frac{\lambda_2}{\gamma} Z_{2t-1} + U_{2t} \qquad (2.7.10)$$

where

$$\begin{array}{lll} Z_{2t} = 0 & \text{if} & \Delta P_t > 0 \\ \quad -\Delta P_t & & \text{otherwise} \end{array}$$

Now combining (2.7.4), (~~1~~2.7.9), and (2.7.10), and taking expectations of both sides, conditional on I_{t-1}, we get

$$P_t^e = \delta_1 (\beta_1' X_{1t}^* - \beta_2' X_{2t}^*) + \delta_2 P_{t-1} + \delta_3 Z_{1t-1} + \delta_4 Z_{2t-1} \qquad (2.7.11)$$

where X_{1t}^* and X_{2t}^* are expected values of X_{1t} and X_{2t} respectively and

$$\delta_1 = \frac{\gamma}{1 + \gamma(\lambda_1 - \alpha_1)} \qquad \delta_2 = \frac{1}{1 + \gamma(\lambda_1 - \alpha_1)}$$

$$\delta_3 = \frac{\cancel{\delta_2}\ \alpha_2}{1 + \gamma(\lambda_1 - \alpha_1)} \qquad \delta_4 = \frac{-\lambda_2}{1 + \gamma(\lambda_1 - \alpha_1)}$$

Equation (2.7.11) expresses the unobservable P_t^e in terms of observables. Further, note that in view of (2.7.3) and (2.7.4), we may write

$$D_t = Q_t + \frac{1}{\gamma} Z_{1t}$$

and

$$S_t = Q_t + \frac{1}{\gamma} Z_{2t}$$

Therefore, equations (2.7.9) and (2.7.10) can be rewritten as

$$Q_t = \beta_1' X_{1t} + \alpha_1 P_t - \frac{1}{\gamma} Z_{1t} + \frac{\alpha_2}{\gamma} Z_{1t-1} + U_{1t} \qquad (2.7.12)$$

and

$$Q_t = \beta_2' X_{2t} + \lambda_1 P_t^e - \frac{1}{\gamma} Z_{2t} + \frac{\lambda_2}{\gamma} Z_{2t-1} + U_{2t} \qquad (2.7.13)$$

The system consisting of (2.7.11), (2.7.12), and (2.7.13) may now be estimated by a two-stage estimation technique, as suggested by Amemiya (1974).

We have to substitute the expression for P_t^e as obtained in (2.7.11). As for X_{1t}^* and X_{2t}^* we can proceed in the same manner as in Section 2.2. This introduces additional cross equation restrictions.

There are two alternatives to use. One is to use nonlinear least squares methods, and the other is to use the ML method. Though the likelihood function is tractable and can be written down in an analogous fashion to thatof Laffont and Monfort, the expressions get somewhat more complicated because of the substitution of equation (2.7.11). The properties of the non-linear least squares estimators compared to the maximum likelihood estimators have been discussed in Lee (1977) and since these properties carry through in these models, there is no further point in discussing them here. Though the non-linear least squares procedure is to be favored in this model, the derivation of the likelihood function will be presented here for completeness of discussion.

Let $h_t(Q_t,P_t)$ be the joint density function of the endogenous variables Q_t and P_t. The likelihood function of the model is then given by

$$L = \prod_{t=1}^{T} h_t(Q_t,P_t) \tag{2.7.14}$$

We now derive the explicit form for $h_t(Q_t,P_t)$.

Let us define an indicator function $I_A(x)$ which assumes the value 1 if $x \in A$ and 0 otherwise. The model comprising of (2.7.11), (2.7.12), and (2.7.13) can then be written in the following form

$$\begin{bmatrix} U_{1t} \\ \\ U_{2t} \end{bmatrix} = I_{R^+}(\Delta P_t) I_{R^+}(\Delta P_{t-1}) \begin{bmatrix} 1 & -\alpha_1 + \frac{1}{\gamma} \\ \\ 1 & 0 \end{bmatrix} \begin{bmatrix} Q_t \\ \\ P_t \end{bmatrix}$$

$$- \begin{bmatrix} \beta_1'X_{1t} + \frac{1}{\gamma} P_{t-1} + \frac{\alpha_2}{\gamma} Z_{1t-1} \\ \beta_2'X_{2t} + \lambda_1 P_t^e \end{bmatrix}$$

$$+ I_{R^+}(\Delta P_t) I_{R^-}(\Delta P_{t-1}) \begin{bmatrix} 1 & -\alpha_1 + \frac{1}{\gamma} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} Q_t \\ P_t \end{bmatrix}$$

$$- \begin{bmatrix} \beta_1'X_{1t} + \frac{1}{\gamma} P_{t-1} \\ \beta_2'X_{2t} + \lambda_1 P_t^e + \frac{\lambda_2}{\gamma} Z_{2t-1} \end{bmatrix}$$

$$+ I_{R^-}(\Delta P_t) I_{R^+}(\Delta P_{t-1}) \begin{bmatrix} 1 & -\alpha_1 \\ 1 & -\frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} Q_t \\ P_t \end{bmatrix}$$

$$- \begin{bmatrix} \beta_1'X_{1t} + \frac{\alpha_2}{\gamma} Z_{1t-1} \\ \beta_2'X_{2t} + \lambda_1 P_t^e - (\frac{1}{\gamma}) P_{t-1} \end{bmatrix}$$

$$+ I_{R-}(\Delta P_t) I_{R-}(\Delta P_{t-1}) \begin{pmatrix} 1 & -\alpha_1 \\ 1 & -\frac{1}{\gamma} \end{pmatrix} \begin{pmatrix} Q_t \\ P_t \end{pmatrix}$$

$$- \begin{pmatrix} \beta_1' X_{1t} \\ \beta_2' X_{2t} + \lambda_1 P_t^e + (\frac{\lambda_2}{\gamma})\ Z_{2t-1} - (\frac{1}{\gamma})\ P_{t-1} \end{pmatrix} \qquad (2.7.15)$$

The piecewise linear mapping is one to one if and only if the determinants of the two matrices

$$\begin{pmatrix} 1 & -\alpha_1 + \frac{1}{\gamma} \\ 1 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & -\alpha_1 \\ 1 & -\frac{1}{\gamma} \end{pmatrix}$$

are of the same sign which is obviously true (since they have the same determinant). These conditions ensure the existence of a well-defined reduced form and are derived from the coherency theorem proved by Gourieroux, Laffont, and Monfort (1980b).

The joint density function $h_t(Q_t, P_t)$ can, in that case, be easily obtained by inverse mapping of equation (2.7.15) above, i.e.,

$$h_t(Q_t, P_t) = (\frac{1}{\gamma} - \alpha_1) I_{R+}(\Delta P_t) I_{R+}(\Delta P_{t-1})$$

$$
\text{x}g\left[\begin{pmatrix} Q_t \\ \\ P_t \end{pmatrix} - \begin{pmatrix} \beta_1' X_{1t} + \frac{1}{\gamma} P_{t-1} + \frac{\alpha_2}{\gamma} Z_{1t-1} \\ \\ \beta_2' X_{2t} + \lambda_1 P_t^e \end{pmatrix}\right]
$$

$$
+ \left(\frac{1}{\gamma} - \alpha_1\right) I_{R+}(\Delta P_t) I_{R-}(\Delta P_{t-1})
$$

$$
\text{x}g\left[\begin{pmatrix} Q_t \\ \\ P_t \end{pmatrix} - \begin{pmatrix} \beta_1' X_{1t} + \frac{1}{\gamma} P_{t-1} \\ \\ \beta_2' X_{2t} + \lambda_1 P_t^e + \left(\frac{\lambda_2}{\gamma}\right) Z_{2t-1} \end{pmatrix}\right]
$$

$$
+ \left(\frac{1}{\gamma} - \alpha_1\right) I_{R-}(\Delta P_t) I_{R+}(\Delta P_{t-1})
$$

$$
\text{x}g\left[\begin{pmatrix} Q_t \\ \\ P_t \end{pmatrix} - \begin{pmatrix} \beta_1' X_{1t} + \frac{\alpha_2}{\gamma} Z_{1t-1} \\ \\ \beta_2' X_{2t} + \lambda_1 P_t^e - \left(\frac{1}{\gamma}\right) P_{t-1} \end{pmatrix}\right]
$$

$$
+ \left(\frac{1}{\gamma} - \alpha_1\right) I_{R-}(\Delta P_t) I_{R-}(\Delta P_{t-1})
$$

$$
\text{x}g\left[\begin{pmatrix} Q_t \\ \\ P_t \end{pmatrix} - \begin{pmatrix} \beta_1' X_{1t} \\ \\ \beta_2' X_{2t} + \lambda_1 P_t^e + \left(\frac{\lambda_2}{\gamma}\right) Z_{2t-1} - \left(\frac{1}{\gamma}\right) P_{t-1} \end{pmatrix}\right] \qquad (2.7.16)
$$

Now, define

$$\psi_1 = \{t: \Delta P_t > 0 \text{ and } \Delta P_{t-1} > 0\}$$

$$\psi_2 = \{t: \Delta P_t > 0 \text{ and } \Delta P_{t-1} < 0\}$$

$$\psi_3 = \{t: \Delta P_t < 0 \text{ and } \Delta P_{t-1} > 0\}$$

$$\psi_4 = \{t: \Delta P_t < 0 \text{ and } \Delta P_{t-1} < 0\}$$

Then one can finally write the likelihood function (2.7.14) in the following form

$$L = (\frac{1}{\gamma} - \alpha_1)^T$$

$$\times \prod_{t\varepsilon\psi_1} g \left[\begin{pmatrix} Q_t \\ \\ P_t \end{pmatrix} - \begin{pmatrix} \beta_1' X_{1t} + \frac{1}{\gamma} P_{t-1} + \frac{\alpha_2}{\gamma} Z_{1t-1} \\ \\ \beta_2' X_{2t} + \lambda_1 P_t^e \end{pmatrix} \right]$$

$$\times \prod_{t\varepsilon\psi_2} g \left[\begin{pmatrix} Q_t \\ \\ P_t \end{pmatrix} - \begin{pmatrix} \beta_1' X_{1t} + \frac{1}{\gamma} P_{t-1} \\ \\ \beta_2' X_{2t} + \lambda_1 P_t^e + (\frac{\lambda_2}{\gamma}) Z_{2t-1} \end{pmatrix} \right]$$

$$\times \prod_{t\varepsilon\psi_3} g \left[\begin{pmatrix} Q_t \\ \\ P_t \end{pmatrix} - \begin{pmatrix} \beta_1' X_{1t} + \frac{\alpha_2}{\gamma} Z_{1t-1} \\ \\ \beta_2' X_{2t} + \lambda_1 P_t^e - (\frac{1}{\gamma}) P_{t-1} \end{pmatrix} \right]$$

$$x \prod_{t\varepsilon\psi_4} g \left[\begin{pmatrix} Q_t \\ \\ P_t \end{pmatrix} - \begin{pmatrix} \beta_1' X_{1t} \\ \\ \beta_2' X_{2t} + \lambda_1 P_t^e + (\frac{\lambda_2}{\gamma}) Z_{2t-1} - (\frac{1}{\gamma}) P_{t-1} \end{pmatrix} \right] \quad (2.7.17)$$

In all these expressions one needs to substitute the expression for P_t^e as obtained from equation (2.7.11), (2.2.7), and (2.2.8).

2.8 A Model with Inventories

One other problem that has been ignored till now is that of inventories. Spillovers on the supply side are better accounted for by introducing inventories explicitly. We will, however, retain spillovers of past unsatisfied demand in the current demand function as postulated in equation (2.7.1). Dagenais (1980) considers inventories and spillovers in the demand function and suggests a limited information estimation method. However, he does not introduce expectations which will be done in what follows.

The literature on inventories often postulates that each firm has a desired level of inventory (see e.g., Lovell, 1961; Hay, 1970; Feldstein and Auerbach, 1976; etc.). If the inventory in a period is too high, the firm would have to incur high storage cost. On the other hand, if the inventory is too low, then in the event of a high demand, there will be high "stock-out" cost (i.e., the disappointed customers may turn ot other firms). Each firm therefore desires to maintain a target level of inventory based on these considerations.

The unobservable target level of inventories is usually assumed to depend on the expected level of future sales, i.e.,

$$I_t^* = \theta_0 + \theta_1 Q_t^e + U_{3t} \tag{2.8.1}$$

where

I_t^* = desired level of inventory in period t

$Q_t^e = E(Q_t | I_{t-1})$ = expected level of future sales

Our model now essentially consists of equations (2.7.1), (2.7.3), (2.7.4), and a production function of the following kind.

$$Y_t = \beta_2' X_{2t} + \lambda_1 P_t^e + \lambda_2 (INV_{t-1} - I_t^*) + U_{2t} \tag{2.8.2}$$

Since the quantity supplied during a given period consists of Y_t, the production plus the inventories in hand, the supply function now becomes

$$S_t = \beta_2' X_{2t} + \lambda_1 P_t^e + \lambda_2' INV_{t-1} + \lambda_3' I_t^* + U_{2t} \tag{2.8.3}$$

where

$$\lambda_2' = (1 + \lambda_2) \quad \text{and} \quad \lambda_3' = -\lambda_2$$

Note that the model considered in section 2.7 is a special case of this model $I_t^* = 0$ while in this model I_t^* is given by equation (2.8.1).

The only problem of estimation here is that the data on Q_t^e, the expected sales, are not available. Some of the previous studies on inventories (Pashigan, 1965; Orr, 1967; Lovell, 1967; Joyce, 1973) have suggested using actual data on ex ante sales expectations such as those collected by the U.S. Department of Commerce. A number of researchers (e.g., Robinson, 1959; Ball and Drake, 1963; Trivedi, 1973; and Grossman, 1973) postulated the future expected sales to be a distributed lag over the past sales. Johnston (1961) used Ferber's law of expectations to obtain a structural proxy for Q_t^e. Several studies (e.g., Darling, 1959; Bridge and Wilson, 1961) also employed a "naive" model of expectations owing to its simplicity, which assume $Q_t^e = Q_{t-1}$. A priori, it cannot be determined which of these structural proxies is a better measure of the expected sales and clearly there is some ad hocism in this area.

Since the expectation mechanism in our analysis is assumed to be "rational", the expected sales are to be obtained in such manner that they coincide with the predictions of the model. For that purpose, let us proceed as follows.

In view of (2.8.1) and (2.8.3), the supply function can be rewritten as

$$S_t = \tilde{\beta}_2' X_{2t} + \psi_1 P_t^e + \psi_2 INV_{t-1} + \psi_3 Q_t^e + U_{4t} \qquad (2.8.4)$$

where

$$\psi_1 = \lambda_1; \ \psi_2 = \lambda_2'; \ \psi_3 = \lambda_3' \theta_1$$

$$U_{4t} = U_{2t} + \lambda_3' U_{3t}$$

and $\tilde{\beta}$ is the same as β_2 except for the constant term which now involves $\lambda_3' \theta_0$.

Now proceeding in the same manner as in section 2.7, we obtain

$$P_t^e = \delta_1(\beta_1'X_{1t}^* - \beta_2'X_{2t}^*) + \delta_2 P_{t-1} + \delta_3 Z_{1t-1} + \delta_4 INV_{t-1} + \delta_5 Q_t^e \tag{2.8.5}$$

where

$$\delta_1 = \frac{\gamma}{1 + \gamma(\psi_1 - \alpha_1)} \qquad \delta_2 = \frac{1}{1 + \gamma(\psi_1 - \alpha_1)}$$

$$\delta_3 = \frac{\alpha_2}{1 + \gamma(\psi_1 - \alpha_1)} \qquad \delta_4 = \frac{-\psi_2\gamma}{1 + \gamma(\psi_1 - \alpha_1)}$$

$$\delta_5 = \frac{-\psi_3\gamma}{1 + \gamma(\psi_1 - \alpha_1)}$$

that

$$D_t = Q_t + \frac{1}{\gamma} Z_{1t}$$

and

$$S_t = Q_t + \frac{1}{\gamma} Z_{2t}$$

we may rewrite the demand and supply equations as

$$Q_t = \beta_1'X_{1t} + \alpha_1 P_t - \frac{1}{\gamma} Z_{1t} + \frac{\alpha_2}{\gamma} Z_{1t-1} + U_{1t} \tag{2.8.6}$$

and

$$Q_t = \tilde{\beta}_2 X_{2t} + \psi_1 P_t^e - \frac{1}{\gamma} Z_{2t} + \psi_2 INV_{t-1} + \psi_3 Q_t^e + U_{4t} \tag{2.8.7}$$

To obtain Q_t^e, let us take expectations of both sides of equation (2.8.6) condition on I_{t-1}. Then,

$$Q_t^e = \beta_1' X_{1t} + \alpha_1 P_t^e - \frac{1}{\gamma} E(Z_{1t} | I_{t-1}) + \frac{\alpha_2}{\gamma} Z_{1t-1} \qquad (2.8.8)$$

Therefore, to obtain Q_t^e, one has to find out the expectation of the non-linear term Z_{1t} conditional on I_{t-1}. For this purpose, we employ a procedure similar to the "conditional expectations" technique developed by Bowden (1978).

Let n denote the density function and N the distribution function of the normal distribution. It is easy to observe that the distribution on $P_t - P_{t-1}$ conditional on I_{t-1} is then (m_t, σ^2) where

$$m_t = \delta_1(\beta_1' X_{1t} - \tilde{\beta}_2' X_{2t}) + (\delta_2 - 1) P_{t-1} + \delta_3 X_{1t-1}$$

$$+ \delta_4 INV_{t-1} + \delta_5 Q_t^e$$

and

$$\sigma^2 = \text{Var}\,[\gamma(U_{1t} - U_{4t})]$$

assuming that U_{1t} and U_{4t} are normally distributed.

Therefore,

$$E(Z_{1t} | I_{t-1}) = \int_0^\infty x\, n(x;\, m_t,\, \sigma^2)\, dx$$

$$= m_t N(m_t,\, 0,\, \sigma^2) + \frac{\sigma}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{m_t}{\sigma}\right)^2} \qquad (2.8.9)$$

(The interested reader is referred to Bowden (1978) for details.)

From the practical point of view, we calculate $\hat{m}_t$ as $\hat{P}_t - P_{t-1}$ where $\hat{P}_t$ is the fitted value from the regression of P_t on P_{t-1}, X_{1t}, X_{2t}, Z_{1t-1}, INV_{t-1}, and Q^e_t. Since the data on Q^e_t are not available, we propose to use Q_{t-1} (or, a distributed lag specification) as an initial proxy for Q^e_t. $\hat{\sigma}$ is obtained as the estimated standard error from this regression. Once $\hat{m}_t$ and $\hat{\sigma}^2$ have been determined, one may obtain $E(Z_{1t}|I_{t-1})$ from (2.8.9) and substitute them in (2.8.8) to derive new estimates of Q^e_t. For practical purposes, we regress Q_t on X_{1t}, $\hat{P}_t$, $E(Z_{1t}|I_{t-1})$ and INV_{t-1} (observe that $\hat{P}_t$ is a valid instrument for P^e_t) and estimate Q^e_t by $\hat{Q}_t$ where $\hat{Q}_t$ is the fitted value from the regression equation. Use them in (2.8.8) and (2.8.7) to estimate the parameters by maximizing the likelihood function and also obtain the maximum value of the likelihood function. Now, use the $\{Q^e_t\}$ series to re-estimate $E(Z_t|I_{t-1})$ from (2.8.9) and use it to re-estimate Q^e_t. Use them to estimate the parameters of (2.8.6) and (2.8.7) again by maximizing the likelihood function. Continue this process until the difference between two successive maxima of the likelihood functions becomes less than some prespecified value, or place an upper bound on the number of iterations to be performed.

An alternative procedure that is computationally simpler would be to estimate Q^e_t as above by $\hat{Q}_t$ where $\hat{Q}_t$ is the fitted value from the regression of Q_t on X_{1t}, $\hat{P}_t$, $E(Z_{1t}|I_{t-1})$, and INV_{t-1}. Use the Q^e_t series to re-estimate $E(Z_t|I_{t-1})$ from (2.8.9) and use to re-estimate Q^e_t. At each stage, obtain the residual variance of the regression equation of Q_t on X_{1t}, $\hat{P}_t$, $E(Z_{1t}|I_{t-1})$, and INV_{t-1}. Repeat this process until the difference between two successive error variances becomes less than some prespecified value (or stop after a certain number of iterations or whichever is earlier). Once the $\{Q_t^e\}$ series have been obtained, use them to estimate the parameters of (2.8.6) and (2.8.7) by a maximum likelihood procedure.

2.9 Tests for Disequilibrium

In the framework of partial adjustment of prices that we have been considering, the test for disequilibrium is simply based on the adjustment coefficient. If the adjustment equation is

$$P_t - P_{t-1} = \lambda(P_t^* - P_{t-1})$$

then a test for disequilibrium is a test for the hypothesis that $\lambda = 1$.

This can be shown to imply the hypothesis that $\eta = \frac{1}{\gamma} = 0$ where γ is the parameter in equation (2.2.4). Note that in our discussion, the parameter γ occurs in P_t^e as given by equation (2.2.6) and in equations (2.3.5) and (2.3.6). The proper way to test the hypothesis that $\eta = \frac{1}{\gamma} = 0$ is to reparameterize these equations in terms of $\eta = \frac{1}{\gamma}$ before the estimation is done.

There is only one additional problem and it is that the model is unstable for $\eta < 0$. Thus the null hypothesis $\eta = 0$ lies on the boundary of the set of admissible values of η.

Gourieroux, Holly and Monfort (1982) suggest using the upper 2α percentage point of the χ^2 distribution in order that the test may have a significance level of α in large samples. Upcher (1982) developed a Lagrange multiplier or score statistic. The score test is not affected by the boundary problem (Moran, 1971) and only requires estimation of the constrained model, i.e., the model under the hypothesis of equilibrium. This test is therefore computationally much simplier than either LR or Wald test and in case the null hypothesis of equilibrium is accepted, one can avoid the burdensome method of estimating the disequilibrium model. Upcher's analysis shows that the score test statistic is identical for both stochastic and non-stochastic specification of the price-adjustment equation. The advantage of this result is that it encompasses a broad spectrum of alternatives. But, in case the null hypothesis of equilibrium is rejected, a range of alternative specifications for the disequilibrium model is possible.

However, a major objection to the use of the Lagrange multiplier procedure is that it ignores the one-sided nature of the alternative and, therefore, is likely to result in a test with low power compared to LR or Wald test procedures. In fact, Bartholomew (1961) pointed out that the power of such tests that do not account for the one-sidedness of the alternative may be reduced by a factor of two, in some cases, relative to the procedures that take this aspect into account.

This issue has been recently addressed by Rogers (1983) who has proposed a test statistic that is asymptotically equivalent under the null hypothesis and a sequence of local alternatives, to the LR and Wald statistics, and which has the same computational advantage over these statistics as does the Lagrange multiplier statistic over the LR and Wald statistics in case of the usual two-sided alternatives.

Similar re-parameterization can be done in the case of the other models considered in sections 2.4, 2.6, 2.7 and 2.8. Since this is straightforward the details will not be pursued here. In the special case where the exogenous variables are known at time t, one can merely substitute P_t for P_t^e and proceed with the usual procedures of testing for disequilibrium in models with no expectational variables.

2.10 Conclusion

The present chapter discusses the disequilibrium model where the source of disequilibrium is imperfect adjustment of prices to their market equilibrating levels. This is the model termed by Fair and Jaffee (1972) as the "Quantitative" model for which Amemiya (1974) discusses the maximum likelihood and two-stage estimation methods. The partial adjustment interpretation for this model is discussed in Maddala (1983b).

In this chapter we discuss various factors in this model that have been ignored in the existing econometric literature. These factors are, however, important in any empirical application of this model. The extensions we have specifically discussed are as follows:

(a) The introduction of expectational variables in the model.

(b) The problem of serially correlated errors.

(c) The problem of expectations of future variables.

(d) The problem of spillovers of previous period's excess demand and excess supply. This problem was analyzed by Laffont and Monfort (1979) but we extend their analysis to the case of models with expectational variables.

(e) Models with inventories.

Finally, we also discuss the modifications necessary in the tests for disequilibrium suggested in the current econometric literature when expectational variables are introduced.

In the next chapter we will discuss the estimation of another class of disequilibrium models, viz., models with controlled prices. We will discuss the estimation problems when there are expectational variables in these models.

CHAPTER 3

METHODS OF ESTIMATION FOR MODELS OF MARKETS WITH CONTROLLED PRICES UNDER RATIONAL EXPECTATIONS

3.1 Introduction

In the previous chapter, we have discussed models in which disequilibrium occurred due to imperfect adjustment to the market equilibrating price. However, disequilibrium may also arise in a market in which the prices are controlled. The case of controlled price is quite different from the case of fixed prices. In the event of fixed prices, the market is in disequilibrium for all time periods, except for a set of measure zero. With controlled prices the market is sometimes in equilibrium while in disequilibrium at other times.

There are many instances of markets with controlled prices. For example, exogenously determined price ceilings are often imposed on the markets for natural gas and therefore in such markets, we can have excess demand or equilibrium points while for markets of agricultural commodities with price supports, there would be excess supply or equilibrium points. In a commodity futures market, trading takes place within exogenously fixed price limits. If the price hits the lower limit, there is excess supply and when the price hits the upper limit, there is excess demand. Otherwise, we have an equilibrium. Thus, in all these markets there are exogenously fixed price limits due to which some observations refer to equilibrium points while others refer to disequilibrium points.

Consider the following model:

$$D_t = \beta_1' X_{1t} + \alpha_1 P_t + U_{1t} \tag{3.1.1}$$

$$S_t = \beta_2' X_{2t} + \alpha_2 P_t + U_{2t} \tag{3.1.2}$$

where D_t and S_t are the quantity demanded and supplied, respectively, P_t is the price, X_{1t} and X_{2t} are sets of exogenous variables, U_{1t} and U_{2t} are residuals that are serially uncorrelated over time. Assume further that P_t is controlled to lie between $\overline{P}_{1t}$ and $\overline{P}_{2t}$, i.e., $\overline{P}_{1t} < P_t < \overline{P}_{2t}$, where $\overline{P}_{1t}$ and $\overline{P}_{2t}$ are exogenously fixed price limits.

Observe that if the equilibrating price is within the specified limits, there is equilibrium and Q_t, the quantity transacted, is given by $D_t = S_t = Q_t$. In this case, both P_t and Q_t are endogenous variables. If the price falls beyond the specified price limits, there will be disequilibrium and P_t is exogenous. Thus, it is essentially a switching simultaneous system in which P_t is sometimes endogenous and sometimes exogenous. There have been some studies involving shifts between the endogenous and exogenous variables (e.g., Barten and Bronsard, 1970; Richard, 1978; Davidson, 1978; etc.). But in all these studies, the switching between regimes is considered exogenous rather than endogenous as in our model where a switch is produced by controls in the market equilibrating price.

The problem has been analyzed by Maddala (1983a), who classified the observations into three regimes.

Regime 1: Define $\psi_1 = \{t: \bar{P}_{1t} < P_t < \bar{P}_{2t}\}$

The set ψ_1 corresponds to equilibrium points for which Q_t and P_t are both endogenous.

Regime 2: Define $\psi_2 = \{t: P_t \leq \bar{P}_{1t}\}$

The set ψ_2 is characterized by excess supply points for which $Q_t = D_t$, $S_t \geq Q_t$, and $\bar{P}_{1t}$ is exogenous.

Regime 3: Define $\psi_3 = \{t: P_t \geq \bar{P}_{2t}\}$

For all $t \in \psi_2$, $Q_t = S_t$ and $D_t \geq Q_t$. Further $\bar{P}_{2t}$ is an exogenous variable.

The appropriate likelihood function for the model is then derived as follows:

$$L = \prod_{\psi_1} f(Q_t,P_t) \prod_{\psi_2} \int_{Q_t}^{\infty} g_1(Q_t,S_t)dS_t \prod_{\psi_3} \int_{Q_t}^{\infty} g_2(D_t)dD_t \tag{3.1.3}$$

where $f(Q_t,P_t)$ is the joint density of Q_t and P_t derived from the joint density of U_{1t} and U_{2t} as in any simultaneous equations model and g_1 and g_2 are the joint densities of D_t and S_t derived from (3.1.1) and (3.1.2) after substituting $P_t = \bar{P}_{1t}$ and $P_t = \bar{P}_{2t}$, respectively. It may be noted that the jacobian of transformation for $f(Q_t,P_t)$ is $|\alpha_1 - \alpha_2|$

which is likely to be non-zero since α_1 and α_2 have opposite signs. The jacobians of transformation for g_1 and g_2 are, of course, unity.

The paper by Maddala (1983a) also discusses a two-stage estimation procedure for the above model and outlines a procedure for estimating the market equilibrating price if controls are removed.

We will now examine the problem of estimating the above model when the supply function is assumed to be dependent on expected price rather than on actual price. The expectation mechanism is, as usual, assumed to be rational. The estimation in this case would be quite complicated since the expectation mechanism turns out to be determined in an implicit manner unlike in the usual case.

3.2 Rationing Model

Consider the following model with an exogenously set price limit $\bar{P}_t$.

$$D_t = \beta_1' X_{1t} + \alpha_1 P_t + U_{1t} \tag{3.2.1}$$

$$S_t = \beta_2' X_{2t} + \alpha_2 P_t^e + U_{2t} \tag{3.2.2}$$

$$Q_t = D_t = S_t \quad \text{if} \quad P_t < \bar{P}_t \tag{3.2.3}$$

$$Q_t = S_t \quad \text{if} \quad P_t \geq \bar{P}_t \tag{3.2.4}$$

where

D_t is the quantity demanded

S_t is the quantity supplied

Q_t is the quantity transacted

P_t is the price

$\overline{P}_t$ is the exogenously set price ceiling and X_{1t} and X_{2t} are explanatory variables.

Given the observations on P_t and $\overline{P}_t$, we can classify the observations as those belonging to equilibrium and those belonging to excess demand.

Let ψ_1 denote the set of observations with $P_t < \overline{P}_t$. For these we have $Q_t = D_t = S_t$ and P_t is an endogenous variable. Let ψ_2 be the set of observations with $P_t = \overline{P}_t$. For these observations we have $S_t = Q_t$ and $D_t > Q_t$ and for this set P_t is an exogenous variable.

The expectation mechanism is again assumed to be "rational" in the sense that expectations are the same as the predictions of the model contingent on the information set the economic agents are assumed to have. Therefore

$$P_t^e = E(P_t | I_{t-1}) \qquad (3.2.5)$$

where I_{t-1} represents the information set available to the economic agents at time t - 1.

Let π_t denote the probability of equilibrium, i.e.,

$$\pi_t = \text{Prob } [P_t < \overline{P}_t]$$

and P_{1t}^e be the price the economic agents expect to prevail in the market in the event of equilibrium conditional on I_{t-1}. Then,

$$\begin{aligned} P_t^e &= E(P_t | I_{t-1}) \\ &= E(P_t | I_{t-1}, t \varepsilon \psi_1) \text{ Prob}(t \varepsilon \psi_1) + \\ &\quad E(P_t | I_{t-1}, t \varepsilon \psi_2) \text{ Prob}(t \varepsilon \psi_2) \\ &\quad P_{1t}^e \pi_t + \bar{P}_t (1 - \pi_t) \end{aligned} \tag{3.2.6}$$

It is intuitively appealing that the price expected to rule the market at time t contingent on I_{t-1} is a weighted average of the expected price during equilibrium and the ceiling price that prevails during disequilibrium.

Now

$$\begin{aligned} \pi_t &= \text{Prob}[P_t < \bar{P}_t] \\ &= \text{Prob}[\frac{1}{\alpha_1} (\beta_2' X_{2t} + \alpha_2 P_t^e - \beta_1' X_{1t} + U_{2t} - U_{1t}) < \bar{P}_t] \\ &= \text{Prob}[U_{2t} - U_{1t} < (\alpha_1 - \alpha_2)(\bar{P}_t - P_t^*) \\ &\qquad - \alpha_2 \pi_t (P_{1t}^e - \bar{P}_t)] \\ &= \Phi(C_t) \end{aligned} \tag{3.2.7}$$

where

$$P_t^* = \frac{\beta_2' X_{2t} - \beta_1' X_{1t}}{\alpha_1 - \alpha_2}$$

$$C_t = \frac{1}{\sigma} [(\alpha_1 - \alpha_2)(\bar{P}_t - P_t^*) - \alpha_2 \pi_t (P_{1t}^e - \bar{P}_t)]$$

and

$$\sigma^2 = \text{Var } (U_{2t} - U_{1t})$$

$\phi(\cdot)$ and $\Phi(\cdot)$ denote the density function and the distribution function of a standard normal variable.

Now before we can proceed any further with the problem of estimating the model, we have to show that this system of equations determines a unique π_t for given values of the exogenous variables and parameters in the system. For this we prove the following lemma.

Lemma: For given values of exogenous variables and the parameters, the system consisting of equations (3.2.1) through (3.2.7) determines a unique solution for π_t in the open interval (0,1).

Proof: Since by definition

$$P^e_{1t} = E(P_t | I_{t-1}, P_t < \bar{P}_t)$$

we have

$$P^e_{1t} < \bar{P}_t \tag{3.2.8}$$

Also, since α_2 is expected to be positive, the value of C_t at $\pi_t = 1$ is greater than the value of C_t at $\pi_t = 0$. Furthermore, note that

$$\frac{dC_t}{d\pi_t} = -\alpha_2 (P^e_{1t} - \bar{P}_t) - \alpha_2 \pi_t \frac{dP^e_{1t}}{d\pi_t}$$

Therefore, in view of (5.2.8) and since $\frac{dP^e_{1t}}{d\pi_t} < 0$, it follows that

$$\frac{dC_t}{d\pi_t} > 0$$

Thus $\phi(C_t)$ is a monotonic function of π_t starting at a value > 0 when $\pi_t = 0$ and ending at a value < 1 when $\pi_t = 1$. Hence, for given values of exogenous variables and the parameters in the system, the equation (3.2.7) admits a unique solution for π_t in the open interval (0,1). This concludes the proof of the lemma.

To obtain P^e_{1t}, we take expectations of (3.2.3) conditional on I_{t-1} and $P_t < \bar{P}_t$ which gives

$$\begin{aligned} \alpha_1 P^e_{1t} + \beta_1' X_{1t} &= \alpha_2 P^e_t + \beta_2' X_{2t} + E(U_{2t} - U_{1t} | U_{2t} - U_{1t} < \sigma C_t) \\ &= \alpha_2 P^e_t + \beta_2' X_{2t} - \sigma \frac{\phi(C_t)}{\Phi(C_t)} \qquad (3.2.9) \end{aligned}$$

Substituting (3.2.6) in (3.2.9), we have

$$\alpha_1 P^e_t + \beta_1' X_{1t} = \alpha_2 [P^e_{1t}\pi_t + (1 - \pi_t)\bar{P}_t] + \beta_2' X_{2t} - \sigma \frac{\phi(C_t)}{\Phi(C_t)}$$

On simplification, we get

$$P^e_{1t} = \frac{1}{(\alpha_1 - \alpha_2 \pi_t)} [\alpha_2 (1 - \pi_t)\bar{P}_t + (\alpha_1 - \alpha_2) P^*_t - \sigma \frac{\phi(C_t)}{\Phi(C_t)}] \qquad (3.2.10)$$

Note that, unlike the usual case, the rational expectation of P_t does not have a closed form solution under controlled prices. The solution is implicitly given by equations (3.2.10) and (3.2.11). However, we can proceed with the estimation by approximating P^e_t by a non-linear function of the exogenous variables X_{1t}, X_{2t}, and $\bar{P}_t$. Once P^e_t has been determined in terms of the parameters and exogenous variables, the model is similar to the one analyzed by Maddala (1983a) and the same maximum likelihood and two-stage estimation procedures are applicable.

3.3 Spillover Effects in Models with Controlled Prices

One important question that arises in the case of models with controlled prices is what happens to the excess demand or excess supply. In the case of agricultural commodities with price support programs, the government absorbs the excess supply by outright purchases. We deal with this problem in chapters 4 and 5. In the case of price controls, for instance control on natural gas prices, controls cause consumers to shift to substitute commodities. The excess demand spills over into the markets for substitute commodities. These other markets will be in equilibrium if they are not subject to price controls. The literature on multimarket disequilibrium models by Ito (1980) and Gourieroux, Laffont, and Monfort (1980a) has not considered this type of problem and hence we shall analyze this in some detail. We consider two commodities which are "substitutes" (say natural gas and coal), one of which is subject to price controls. Further, we assume that the other market is always in equilibrium. We thus assume that the other market completely absorbs the excess demand from the market that is subject to price controls. Hence, we can ignore spillovers of past unsatisfied demand from the market subject to price controls.

Our model is

$$D_{1t} = \alpha_0' X_{1t} + \alpha_1 P_{1t} + \alpha_2 P_{2t} + U_{1t} \qquad (3.3.1)$$

$$S_{1t} = \beta_0' X_{2t} + \beta_1 P_{1t} + U_{2t} \qquad (3.3.2)$$

$$Q_{1t} = \text{Min } (D_{1t}, S_{1t}) \qquad (3.3.3)$$

$$Q_{1t} = D_{1t} = S_{1t} \quad \text{if} \quad P_{1t} < \overline{P}_{1t} \qquad (3.3.4)$$

$$Q_{1t} = S_{1t} \quad \text{if} \quad P_{1t} \geq \overline{P}_{1t} \qquad (3.3.5)$$

$$D_{2t} = \gamma_0' Z_{1t} + \gamma_1 P_{2t} + \gamma_2 P_{1t} + \lambda (D_{1t} - S_{1t}) + U_{3t} \qquad (3.3.6)$$

$$S_{2t} = \delta_0' Z_{2t} + \delta_2 P_{2t} + U_{4t} \qquad (3.3.7)$$

$Q_{2t} = D_{2t} = S_{2t}$ i.e., the second market is always in equilibrium.

Here

D_{it} = quantity demanded in the ith market

S_{it} = quantity supplied in the ith market

P_{it} = price in the ith market

$\overline{P}_{1t}$ is the exogenously set price ceiling in the first market

X_{1t}, X_{2t}, Z_{1t}, Z_{2t} are vectors of exogenous variables.

$i = 1,2$

Define

$$\psi_1 = t: \; P_{1t} < \overline{P}_{1t}$$

$$\psi_2 = t: \; P_{1t} \geq \overline{P}_{1t}$$

We will now outline a two-stage estimation procedure to obtain consistent estimates of the parameters of the model above.

In the set ψ_1, we have the usual simultaneous equations model with Q_{1t}, Q_{2t}, P_{1t}, P_{2t} as the endogenous variables and the reduced form for P_{1t} and P_{2t} could be written as

$$P_{1t} = \pi_1 X_t + W_{1t} \tag{3.3.8}$$

$$P_{2t} = \pi_2 X_t + W_{2t} \tag{3.3.9}$$

where X_t includes all the exogenous variables in the model and $(W_{1t}, W_{2t}) \sim \text{IID}\ (\underset{\sim}{0}, \Sigma_w)$.

In the set ψ_2, $P_{1t} (= \overline{P}_{1t})$ is an exogenous variable and the reduced form of P_{2t} could be written as

$$P_{2t} = \pi_3 X_t + W_{3t} \text{ with } W_{3t} \sim \text{IID}\ (0, \sigma^2_{w_3}) \tag{3.3.10}$$

Therefore,

$$\begin{aligned} P_{1t} &= \pi_1 X_t + W_{1t} \quad \text{if} \quad t \in \psi_1 \\ &= \overline{P}_{1t} \text{ otherwise} \end{aligned} \tag{3.3.11}$$

and

$$\begin{aligned} P_{2t} &= \pi_2 X_t + W_{2t} \quad \text{if} \quad t \in \psi_1 \\ &= \pi_3 X_t + W_{3t} \quad \text{otherwise} \end{aligned} \tag{3.3.12}$$

Then, the parameters π_1 in (3.3.11) can be estimated by the tobit method.

Once we obtain the estimates $\hat{\pi}_1$ of the parameters π, by the tobit method, we can get the estimated values $\hat{P}_{1t}$ of P_{1t}. However, the estimated values $\hat{P}_{1t}$ that should be substituted in the structural equations (3.3.1) and (3.3.2) are for the subset ψ_1. Observe that

$$\begin{aligned} E[P_{1t} \mid t \in \psi_1] &= E[\pi_1 X_t + W_{1t} \mid t \in \psi_1] \\ &= E[\pi_1 X_t + W_{1t} \mid \pi_1 X_t + W_{1t} < \overline{P}_{1t}] \\ &= \pi_1 X_t - \sigma_1 \frac{\phi(Z_{1t})}{\Phi(Z_{1t})} \end{aligned}$$

where

$$Z_{1t} = \frac{\overline{P}_{1t} - \pi_1 X_t}{\sigma_1} \qquad (3.3.13)$$

$$\sigma_1^2 = \text{Var}\ (W_{1t}) \qquad (3.3.14)$$

and ϕ and Φ are the density function and distribution function of a standard normal variate. From the tobit estimates $\hat{\pi}_1$ of π_1 and $\hat{\sigma}_1$ of σ_1, we may obtain the estimates $\hat{P}_{1t}$ of P_{1t} from equation (3.3.13).

From (3.3.12), we have

$$\begin{aligned} P_{2t} &= \pi_2 X_t + W_{2t} \quad \text{if} \quad \pi_1 X_t + W_{1t} < \overline{P}_{1t} \\ &= \pi_3 X_t + W_{3t} \quad \text{if} \quad \pi_1 X_t + W_{1t} \geq \overline{P}_{1t} \end{aligned}$$

and such models of binary choice have been estimated by Lee (1979). Following the procedure outlined there, we may obtain the estimates of π_2 and π_3. Once the estimates of π_2 and π_3 have been obtained, it is possible to obtain $E(P_{2t}|t \in \psi_1)$ and $E(P_{2t}|t \in \psi_2)$ as follows.

$$E(P_{2t}|t \in \psi_1) = E(\pi_2 X_t + W_{2t}|\pi_1 X_t + W_{1t} < \overline{P}_{1t})$$
$$= \pi_2 X_t - \frac{\sigma_{12}}{\sigma_1}\frac{\phi(Z_t)}{\Phi(Z_t)} \qquad (3.3.15)$$

where

$$\sigma_{12} = \text{Cov}\ (W_{2t}, W_{1t})$$

Similarly,

$$E(P_{2t}|t \in \psi_2) = \pi_2 X_t + \frac{\sigma_{13}}{\sigma_1}\frac{\phi(Z_t)}{1 - \Phi(Z_t)} \qquad (3.3.16)$$

where

$$\sigma_{13} = \text{Cov}\ (W_{3t}, W_{1t})$$

In view of (3.3.16), we may now obtain $E(D_{1t} - S_{1t}|t \in \psi_2)$ which would serve as estimates of $D_{1t} - S_{1t}$ when $t \in \psi_2$, i.e., when the first market is in disequilibrium.

$$E(D_{1t} - S_{1t}|t \in \psi_2) = E[\alpha_0' X_{1t} - \beta_0' X_{2t} + (\alpha_1 - \beta_1)P_{1t}$$
$$+ \alpha_2 P_t + U_{1t} - U_{2t}|t \in \psi_2]$$
$$= \alpha_0' X_{1t} - \beta_0' X_{2t} + (\alpha_1 - \beta_1)\overline{P}_{1t}$$
$$+ \alpha_2 E(P_{2t}|t \in \psi_2) + E(U_{1t}|t \in \psi_2)$$
$$- E(U_{2t}|t \in \psi_2) \qquad (3.3.17)$$

Now,

$$E(U_{1t}|t \varepsilon \psi_2) = E(U_{1t}|\pi_1 X_t + W_{1t} \geq \bar{P}_{1t})$$

$$= E(U_{1t}|\frac{W_{1t}}{\sigma_1} \geq Z_t)$$

$$= \frac{\sigma_{11}}{\sigma_1} \cdot \frac{\phi(Z_t)}{1 - \Phi(Z_t)} \qquad (3.3.18)$$

where

$$\sigma_{11} = \text{Cov}\ (U_{1t}, W_{1t})$$

and, similarly

$$E(U_{2t}|t \varepsilon \psi_2) = \frac{\sigma_{21}}{\sigma_1} \frac{\phi(Z_t)}{1 - \Phi(Z_t)} \qquad (3.3.19)$$

where

$$\sigma_{21} = \text{Cov}\ (U_{2t}, W_{1t})$$

In view of (3.3.18) and (3.3.19), $E(D_{1t} - S_{1t}|t \varepsilon \psi_2)$ could now be determined from (3.3.17).

Coming next to the structural equations, the demand in market 1 is

$$W_{1t} = \alpha_o' X_{1t} + \alpha_1 P_{1t} + \alpha_2 P_{2t} + U_{1t}$$

and this can be determined from the observations in ψ_1 only. However, we have to make adjustment for the fact that $E(U_{1t}|t \varepsilon \psi_1) \neq 0$. We have

$$E(U_{1t}|t \varepsilon \psi_1) = E(U_{1t}|\frac{W_{1t}}{\sigma_1} < Z_t)$$

$$= -\frac{\sigma_{11}}{\sigma_1} \frac{\phi(Z_t)}{\Phi(Z_t)} \qquad (3.3.20)$$

Therefore, we may write the demand equation for market 1 as follows

$$Q_{1t} = \alpha_0'X_{1t} + \alpha_1P_{1t} + \alpha_2P_{2t} - \frac{\sigma_{11}}{\sigma_1}\frac{\phi(Z_t)}{\Phi(Z_t)} + \eta_{1t} \tag{3.3.21}$$

where the new residual η_{1t} has zero mean. The two-stage least squares method now consists of estimating the parameters in (3.3.21) by OLS after substituting $\hat{P}_{1t}$ for P_{1t} from (3.3.13) and $\hat{P}_{2t}$ for P_{2t} from (3.3.15). Note that $\hat{Z}_t$ is obtained from (3.3.14) using the tobit estimates of $\hat{\pi}$ and $\hat{\sigma}_1$. The estimates can be shown to be consistent.

Coming next to the supply function, we now have observations in both ψ_1 and ψ_2. Again, correcting for the non-zero means of the residuals in the two regimes, we may write

$$E(U_{2t}|t \in \psi_1) = -\frac{\sigma_{11}}{\sigma_1}\frac{\phi(Z_t)}{\Phi(Z_t)} \tag{3.3.22}$$

and,

$$E(U_{2t}|t \in \psi_2) = \frac{\sigma_{21}}{\sigma_1}\frac{\phi(Z_t)}{1 - \Phi(Z_t)}$$

In view of (3.3.19) and (3.3.22), we may write

$$Q_{1t} = \beta_0'X_{2t} + \beta_1P_{1t} - \frac{\sigma_{21}}{\sigma_1}\frac{\phi(Z_t)}{\Phi(Z_t)} + \eta_{2t} \text{ for } t \in \psi_1 \tag{3.3.23}$$

and

$$Q_{1t} = \beta_0'X_{2t} + \beta_1\overline{P}_{1t} + \frac{\sigma_{21}}{\sigma_1}\frac{\phi(Z_t)}{1 - \Phi(Z_t)} + \eta_{2t} \text{ for } t \in \psi_2 \tag{3.3.24}$$

where residuals η_{2t} have zero mean. The two-stage least squares method now consists of estimating the parameters β_0', β_1, $\frac{\sigma_{21}}{\sigma_1}$ by OLS after substituting $\hat{P}_{1t}$ for P_{1t} in (3.3.23) from equation (3.3.13) and $\hat{Z}_t$ for Z_t in (3.3.23) and (3.3.24). The resulting two-stage estimates are consistent.

Next, we consider the second market. The demand equation is

$$Q_{2t} = \gamma_0' X_{1t} + \gamma_1 P_{2t} + \gamma_1 P_{1t} + \lambda (D_{1t} - S_{1t}) + U_{3t}$$

for which we have observations in both ψ_1 and ψ_2. Adjusting for the non-zero means of the residuals in the two regimes, we have

$$E(U_{3t} | t \in \psi_1) = - \frac{\sigma_{31}}{\sigma_1} \frac{\phi(Z_t)}{\Phi(Z_t)} \tag{3.3.25}$$

and,

$$E(U_{3t} | t \in \psi_2) = \frac{\sigma_{31}}{\sigma_1} \frac{\phi(Z_t)}{1 - \Phi(Z_t)} \tag{3.3.26}$$

where

$$\sigma_{31} = \text{Cov }(U_{3t}, W_{1t})$$

Therefore,

$$Q_{2t} = \gamma_0' Z_{1t} + \gamma_1 P_{2t} + \gamma_2 P_{1t} - \frac{\sigma_{31}}{\sigma_1} \frac{\phi(Z_t)}{\Phi(Z_t)} + \varepsilon_{1t} \text{ for } t \in \psi_1 \tag{3.3.27}$$

$$Q_{2t} = \gamma_0' Z_{1t} + \gamma_1 P_{2t} + \gamma_2 \bar{P}_{1t} + \lambda (D_{1t} - S_{1t}) + \frac{\sigma_{31}}{\sigma_1} \frac{\phi(Z_t)}{1 - \Phi(Z_t)}$$

$$+ \varepsilon_{1t} \text{ for } t \in \psi_2 \qquad (3.3.28)$$

where the new residual $\check{\varepsilon}_{1t}$ has zero mean. The two-stage least squares method now consists of estimating the parameters in (3.3.27) and (3.3.28) by OLS after substituting $\hat{P}_{1t}$ from (3.3.13) and P_{2t} from (3.3.15) in equation (3.3.17) and $\hat{P}_{2t}$ from (3.3.16) and $D_{1t} - S_{1t}$ from (3.3.17) in equation (3.3.28). $\hat{Z}_t$ is obtained as before from the tobit estimates of π_1 and σ_1. The two-stage least squares estimates are consistent.

Coming next to the supply function for the second market, we would again have to make a correction for the non-zero means of the residuals in the two regimes. As before, we obtain

$$E(U_{4t} | t \in \psi_1) = - \frac{\sigma_{41}}{\sigma_1} \frac{\phi(Z_t)}{\Phi(Z_t)} \qquad (3.3.29)$$

and,

$$E(U_{4t} | t \in \psi_2) = \frac{\sigma_{41}}{\sigma_1} \frac{\phi(Z_t)}{1 - \Phi(Z_t)} \qquad (3.3.30)$$

where

$$\sigma_{41} = \text{Cov } (U_{4t}, W_{1t})$$

Now, we may write the supply function as

$$Q_{2t} = \delta_0' Z_{2t} + \delta_2 P_{2t} - \frac{\sigma_{41}}{\sigma_1} \frac{\phi(Z_t)}{\Phi(Z_t)} + \varepsilon_{2t} \text{ for } t \in \psi_1 \qquad (3.3.31)$$

and,

$$Q_{2t} = \delta_0' Z_{2t} + \delta_2 P_{2t} + \frac{\sigma_{41}}{\sigma_1} \frac{\phi(Z_t)}{1 - \Phi(Z_t)} + \varepsilon_{2t} \text{ for } t \varepsilon \psi_2 \qquad (3.3.32)$$

where the residual ε_{2t} now has zero mean. The two-stage least squares method now consists of estimating the parameters δ_0', δ_2, and $\frac{\sigma_{41}}{\sigma_1}$ by OLS after substituting P_{2t} from (3.3.15) in equation (3.3.13) and $\hat{P}_{2t}$ from (3.3.16) in equation (3.3.22). $\hat{Z}_t$ is substituted for Z_t in both (3.3.31) and (3.3.32). The resulting two-stage estimates will be consistent.

3.4 Prediction of the Effects of Removal of Price Ceiling

One interesting question from the policy point of view is what the equilibrium price in the first market would have been in periods the price ceiling is operative. Consider the example of the energy market with coal and natural gas as indicated earlier. Suppose, the price control is operative in the natural gas market alone. Then, it may be of interest to the policy makers to know what the equilibrium price for natural gas would be if the price controls are removed. To answer this question, we have to evaluate $E(P_{1t}|t \varepsilon \psi_2)$ which can be computed as follows.

From equation (3.3.8), we have

$$\begin{aligned} E(P_{1t}|t \varepsilon \psi_2) &= \pi_1 X_t + E(W_{1t}|W_{1t} > \bar{P}_{1t} - \pi_1 X_t) \\ &= \pi_1 X_t + \sigma_1 \frac{\phi(Z_t)}{1 - \phi(Z_t)} \qquad (3.4.1) \end{aligned}$$

From the tobit estimates of π and σ_1 we can evaluate (3.4.1).

3.5 Conclusion

In this chapter we have discussed the estimation of the disequilibrium model when disequilibrium is caused by controls on the movement of prices. We have discussed the problems created by the assumption of rational (in the sense of model consistent) expectations. It is not possible in this case to derive an explicit solution for expected price. However, we have suggested an alternative computable procedure that captures the spirit of the rational expectations model.

We have also considered the estimation of a two-market disequilibrium model where excess demand from the market with controlled prices spills over into the other market (which is not subject to any price controls).

In actual practice, the methods of estimation outlined in this chapter need some modifications depending on the structure of the market under consideration. What we have outlined here are the essential elements of the problems that would be encountered in estimating disequilibrium models with controlled prices when we introduce expectational variables and spillovers of excess demand or excess supply. Since it is not possible to discuss all the special problems that need to be taken into account while estimating the models in actual practice, we will confine our attention to one particular model--the case of price supports for corn in the U.S. In the next two chapters, we will discuss this problem and present the empirical results.

CHAPTER 4
ESTIMATION OF A DEMAND AND SUPPLY MODEL WITH PRICE SUPPORTS: AN APPLICATION TO THE U.S. CORN MARKET

4.1 Introduction

An important application of the disequilibrium market models is in the case of agricultural price support programs. Under price support, the market will be in equilibrium when the market equilibrating price is above the support price, but when the market equilibrating price is below the support level, the supply exceeds the demand and there is disequilibrium in the market.

In this chapter, we will examine the corn market in the U.S., which is characterized by price support. Corn is by far the major feed grain in the United States, both in terms of acreage and its value, accounting for nearly two-thirds of the total feedgrain production in the country. Adaptable to an extremely wide range of climatic conditions, the crop is grown in every state in the United States, primarily in the mid-West. The implications of an appropriate model of the corn-market should be of considerable interest, as it is an important link between the crop and the livestock sectors of U.S. agriculture. The object of our analysis here is to compare, inter alia, the magnitude of difference between the disequilibrium methods and the conventional equilibrium approach in terms of supply and demand elasticities. This is accomplished by explicitly accounting for the impact of the price-support programs on both supply and demand. Expectations are introduced in the model through the supply side of the market. In fact, the use of an expected price is quite common in modeling the markets for agricultural commodities as there is a time lag separating the prodcution decisions and the harvested output. Several mechanisms for formation of these price expectations have been suggested in the literature. These include the cobweb or naive (Ezekiel, 1938), the extrapolative (Metzler, 1941), the adaptive (Cagan, 1956, Nerlove, 1958), and the rational expectations hypothesis (Muth, 1961). A number of researchers have also advocated using futures prices in supply analysis (Gardner, 1976; Telser, 1967). An important facet of our analysis will be to examine these alternative schemes of formation of expectation in the corn market in U.S. and assess their relative performance. In doing so, we will explicitly account for the effect of price support on the expectations formed by the producers in the market.

4.2 The Price Program in Corn in the U.S.

Over the period of our analysis, 1949-1981, there has been a plethora of legislations prescribing the modus operandi of government intervention in the corn market in the U.S. Historically, in 1933 Congress declared as its policy to ensure adequate supplies, to stabilize markets and prices and to re-establish prices to farmers at a level that will give agricultural commodities a purchasing power with respect to articles that farmers buy, equivalent to the purchasing power in the base period. The base period for corn was specified as August, 1909 to July, 1914. This paved the way for the enactment of the Agricultural Adjustment Act, 1933 (Triple-A) and its subsequent amendments, in 1935, 1938 and 1941. The primary objective in all these enactments was to prescribe the price relationships which is known as 'parity'.

The price support operations for corn in the U.S. are carried out by the Commodity Credit Corporation (C.C.C.). Under the Agricultural Adjustment Act of 1933, CCC was authorized to provide support loans to the corn producers participating in the support program. The participation in the support program was, however, contingent on the agreement to reduce the acreage under corn. The principal defect of the support program under the Triple-A Act, 1933 was that although the participants in the support program were provided with substantial financial incentives to reduce production and acreage, this enabled the non-participants in the support program to increase their production and acreage to take advantage of a favorable price situation resulting from the operations under price support program. The production control features of the Agricultural Adjustment Act, 1933 were ultimately declared ultra vires of the constitution by the Supreme Court on January 6, 1936. To avoid the complication created by the adverse ruling of the Supreme Court, the Congress enacted the Soil Conservation and Domestic Allotment Act, 1936 providing for soil conserving and soil building payments to cooperating farmers.

The Agriculture Adjustment Act of 1938 put more emphasis on price-support loans provided by the CCC and also integrated the activities of the CCC with acreage allotments and marketing quotas. The Act which is still effective in its amended form, provided for mandatory price support loans on certain 'basic' commodities, which include corn, so that the crops could be stored in time of plenty and returned to the market when supplies became scarce.

The price support in case of corn is primarily provided by means of loans. The loans are made directly to the eligible producers through the County Agricultural Stabilization and Conservation (ASC) offices, or to the approved agricultural cooperative marketing associations on the security of the stored commodities. The loans are 'non recourse' in the sense that the producers are not obligated to repay their loans or make good any decline in the market price of the crop they have put up as collateral. Instead, the producer or the association may deliver, or forfeit the pledged commodity to satisfy the loan obligation without further payment. When the market prices are below the amount due on the loans, the producers would find it advantageous to deliver or forfeit his stock to the CCC rather than pay off their loans to redeem the commodities. However, if the market price is above the support price, the producer would prefer to sell the stock in the market and repay the loans. In addition, reseal loan facilities are also available to the farmers that enable them to extend their loans beyond the initial loan maturity date. The "reseal" is derived from the fact that the crop stored in farms for such loans is again put under seal. These loans aid in timely marketing and are particularly useful in a period of excess supply when the prices are weak. The costs of storage during the reseal period are borne by CCC. Farmers however continue to enjoy the option of redeeming their commodities under reseal loan facilities.

4.3 Different Formulations of the Disequilibrium Model of the Corn Market in the U.S.

Let us consider a simple model structure to examine the effect of the price support program in the corn market in the U.S. Although simple in nature, the model captures the essential flavor of the possibilities of disequilibrium due to the government intervention through price supports in the event of excess supply. The time lag between the planting decision and final output is accounted for by postulating that the quantity produced is determined by the price that the growers expect to prevail in the market at the time the crop is harvested. This expected price is determined by taking into account government price support activities.

The supply equation is of the following form:

$$Q_t = a_0 + a_1 P_t^e + a_2 \hat{YLD}_t + u_{1t} \qquad (4.3.1)$$

where P_t^e is the price expected by the producer and $\hat{YLD}_t$ represents the fitted value obtained by regressing the observed corn yields per harvested acre against sorghum yields, soybean yields and a time trend. The fitted value is used since the production and yield per acre are non-linearly related. Further it is customary in incorporate a proxy for technological progress in the usual specification of the supply function in the literature. This is generally done through a linear trend in the supply specification. Interestingly enough the yield of corn has almost trebled in the U.S. since World War II. Before the Second World War, fertilizer was used on corn only to a very limited extent. The only significant use of fertilizer on corn was in the southeastern part of the United States. Between 1942 and 1952 however, the use of fertilizer went up by 890 percent in Illinois, by 780 percent in Iowa and by 375 percent in Missouri. These states are all large producers of corn. The data on yield reasonably capture this influence. Another variable generally used in the supply specification is a climate index. The difference between acreage planted and harvested is often used as a proxy for weather conditions (e.g., Riboud and Rausser, 1981). This is justified on the ground that any discrepancy between planted and harvested acreage can only be explained by climatic factors since price supports provide an incentive for the growers to harvest as much acreage as allowed. In our approach, the data on yield essentially capture this influence as well since yields are strongly influenced by climatic conditions during the growing season. The yield per harvested acre of sorghum and soybean are therefore used to capture the weather effects common to areas where corn in produced.

Since in our specification, the supply is predetermined, the market demand at the farm level can be represented with price as the relevant dependent variable. In particular, we assume the following specification of the demand function, viz.

$$P_t = b_0 + b_1D_t + b_2DY_t + b_3XP_t + u_{2t} \tag{4.3.2}$$

where D_t = total demand for corn

DY_t = total disposable income

XP_t = quantity exported, commercial plus government

In our approach the quantity demanded, D_t, is determined by aggregating the demand for all uses of the crop. As a result, when the market equilibrating price is above the support price

$$D_t = Q_t + S_{t-1} \tag{4.3.3}$$

i.e., total aggregate demand for corn in the market equals the production, Q_t, plus S_{t-1}, the beginning period stocks in all positions.

However, when the market equilibrating price is below the price support announced by the government, the participants in the support program would prefer to deliver or forfeit their stored commodities on loan to CCC, as they would otherwise incur a loss if their crop is sold in the open market. In such cases, therefore, the aggregate demand for corn is given by

$$D_t = Q_t + S_{t-1} - CG_t \tag{4.3.4}$$

where CG_t denotes the quantity pledged with the CCC by the participants in the support program that is ultimately delivered or forfeited by CCC. CG_t also includes any other purchases made by CCC at the support price. The inclusion of CG_t in our analysis is an important departure from the previous studies of agricultural markets under price supports (e.g., Riboud and Rausser, 1981), where (4.3.4) above is used to obtain aggregate demand even when the equilibrium price is below the support price, thus neglecting the impact of government support programs on the aggregate demand of the market.

The inclusion of the total disposable income, DY_t, in the demand equation for corn can be justified by assuming any functional form of the consumer utility function. Further, it would reasonably capture yet another effect. Much of the demand for corn stems from the livestock sector. Disposable income, DY_t, in the demand equation would account for shifts in the derived demand for corn as the demand for livestock products increases with rising income. The export variable XP_t is introduced in the demand equation as a proxy for foreign income. It also captures the essentials of an exogenous growth in world demand for livestock feed. The expected sign of the estimated coefficient on exports is, therefore, positive.

Our specification of the model of the corn market is obviously a simplified representation of the complexities of the corn market in the U.S. We do not analyze the impact of any other government program variable on the supply or demand functions besides the price support operations. This is primarily because government programs in agriculture have undergone substantial changes over the period of our analysis (see, USDA, 1976) and complexities to account for these changes would only burden the model. In fact, our model captures the essentials of a market under price supports and is fairly robust in answering the questions under investigation. Moreover, it is an attempt to use the data on the government operations through loans and purchases during a period when the equilibrium price falls below the support price, thus envisaging the appropriate role of the government in a price support program.

For structural estimation, we partition the sample observations as follows:

$$\psi_1 = \{t: \; P_t > \overline{P}_t\}$$

ψ_1 therefore consists of observations for which the market price is above the support price announced by the government

$$\psi_2 = \{t: \; P_t \leq \overline{P}_t\}$$

ψ_2 therefore consists of observations for which the observed market price is less than or equal to support price. Theoretically, in a price-support program, the market price should equal the support price if the equilibrium price falls below the price support. However, an examination of the observed market price data in the corn market would reveal that observed market prices were consistently below the support price during 1950's and 1960's. This may happen due to various reasons, such as, producer ineligibility or inability to participate in the price support program, sales by non-producers and government disposition of deteriorating stocks. This point will be discussed in detail in the subsequent chapter.

The joint densities of (Q_t, P_t) in the two regimes are then obtained from equations (4.3.1) and (4.3.2) with

$$D_t = Q_t + S_{t-1} \qquad \text{for } t \in \psi_1$$

$$= Q_t + S_{t-1} - CG_t \text{ for } t \in \psi_2$$

Our model is therefore in a simultaneous equations framework for which asymptotically efficient structural estimates could be obtained by ful-information maximum likelihood techniques.

4.4 Expectations in the Supply Function

The use of expectations in the specification of supply functions is quite common in modeling markets of agricultural commodities. This is because in the case of agricultural commodities, production decisions are required to be taken on the basis of the price expected to prevail at the harvesting time since there is a natural time lag between the productive efforts and final output. As indicated earlier, the expected price that determines the amount of production can be formulated in accordance with alternative schemes of expectations. We would analyze six alternative mechanisms of formation and compare their relative performance to determine how expectations are formed in the corn market in the U.S. We begin our analysis by substituting P_t, the current price in place of P_t^e. Observe that this is equivalent to a rational expectations framework in which the values of exogenous variables for period t are assumed to be known at time t-1. In such events, we essentially have

$$P_t^e = P_t + V_t \qquad (4.4.1)$$

where V_t is uncorrelated with the exogenous variables at time t and therefore one can replace P_t^e by P_t in the supply equation (4.3.1) above. This model is therefore structurally equivalent to one in which there is no price expectations appearing in the supply function. This will be referred to as the 'perfect foresight' model in our analysis.

The next model analyzed assumes a cobweb or 'naive' mechanism of formation of expectations in which the price expected by the growers equals the price of the previous period, i.e., $P_t^e = P_{t-1}$. We will also investigate the extrapolative expectation mechanism as associated with Metzler (1941) in which the price expectations are essentially generated by the autoregressive structure. The ad hoc extrapolative models for price expectations are typically of the following form

$$P_t^e = \sum_i \lambda_i P_{t-i} \tag{4.4.2}$$

The number of lags and the nature of restrictions imposed on the λ_i's in (4.4.2) are usually subjectively determined. Equation (4.4.2) essentially implies that the agents from their expectations based on the past values of the observed market prices. Individual past prices do not exert their influence equally, however. Usually, the more recent the price, the more influence it is likely to have on the measure of expectation of the market price. Observe that the 'cobweb' type of expectation is a special case of the extrapolative expectations model, in which the expectations are formed soley on the basis of the price of the previous period. These mechanisms of formation of expectation, however, ignore the information contained in the structural model as to how prices are determined in the market. Nevertheless, the 'naive' cobweb model has been found to produce better results than more sophisticated schemes of expectation formation such as 'rational' (or 'partially rational' in which a convex combination of cobweb and 'rational' schemes of expectations) in a study of the supply side of the Florida escarole market (Shonkwiler, 1982).

Another possibility of formation of expectation is given by the adaptive expectations hypothesis suggested by Cagan (1956). In this model, the expectations are revised in proportion to the error associated with the previous level of expectations:

$$P_t^e - P_{t-1}^e = \lambda(P_{t-1} - P_{t-1}^e) \tag{4.4.3}$$

where $0 < \lambda < 1$ is the coefficient of expectations. If we solve for the difference equation in P_t^e we have

$$P_t^e = \sum_{t=0}^{\infty} \lambda(1-\lambda)^i P_{t-1-i} \tag{4.4.4}$$

which implies a geometrically declining distributed lag form for the expected prices as a function of all past prices.

The adaptive expectations model has been widely used in agricultural supply analysis. In an exhaustive survey of the literature on agricultural supply analysis, Askari and Cummings (1976) considered more than 600 estimates of supply response to price. The basic model employed in most of these studies was the adaptive expectations mechanism. In our analysis, we combine adaptive expectations along with the possibility of divergence between the market supply and demand in case equilibrium price falls short of the support price. In fact, Nerlove (1979) himself emphasized the need for studying the effect of government intervention in the markets of perennial crops. This is essentially what has been attempted in this exercise.

The difficulty with these expectations schemes is that they all basically imply that expectations are formed in a market on the basis of past prices alone and further that the agents do not explicitly utilize any information about the structure of the model of market in which they are operating. In 1961, Muth advanced the concept of rational expectations according to which expectations are essentially the same as the predictions of the relevant theory. According to Muth, information is costly and therefore no economic system would generally waste it. In rational expectations framework, therefore, expectations are derived contingent on the structure of the system in which the agent is operating. In general, the rational expectations mechanism for the endogenous price variable P_t is given by

$$P_t^e = E(P_t | I_{t-1}) \tag{4.4.4}$$

where I_{t-1} represents the set of information available to the agent at time t-1.

The rational expectation hypothesis essentially maintains that the participants in a market act as if they were solving the supply and demand system in the formation of their expectations. Shonkwiler and Emerson (1982) studied the Florida tomato industry under the hypothesis that the growers form their expectations in accordance with the principle of Muthian rationality. Their empirical findings suggest that tomato growers respond to the market information in its entirety in keeping with the rational expectations hypothesis. We will also examine if the expectations in the corn market in U.S. are consistent with the rational expectations principle. The highly commercial and organized nature of the U.S. corn market makes it reasonable to explore if expectations are formed in this market in a rational manner. To obtain P_t^e under the rational expectations hypothesis in the corn market under price support, let us reconsider our model specified in the following simple form:

$$\text{Supply function:} \quad Q_t = a_0 + a_1 P_t^e + a_2 X_t + U_{1t} \tag{4.4.5}$$

$$\text{Demand function:} \quad P_t = b_0 + b_1 D_t + b_2 Z_t + U_{2t} \tag{4.4.6}$$

$$\text{where} \quad D_t = Q_t + S_{t-1} \qquad \text{if } P_t > \overline{P}_t$$

$$= Q_t + S_{t-1} - CG_t \quad \text{if } P_t \leq \overline{P}_t$$

X_t and Z_t above represent supply and the demand shifters respectively in the market for corn; S_{t-1} represents the beginning stocks in all positions and CG_t is the extent of commodities acquired by CCC under the price support program when the equilibrating price is less than the support price, $\overline{P}_t$, announced by the government.

To derive the rational expectations of P_t in this model, we eliminate Q_t from equation (4.4.6) and obtain the reduced form for P_t as follows:

$$P_t = b_0 + b_1[(a_0 + a_1 P_t^e + a_2 X_t + U_{1t}) + S_{t-1}]$$

$$+ b_2 Z_t + U_{2t} \qquad \text{if } P_t > \overline{P}_t \tag{4.4.7}$$

and

$$P_t = b_0 + b_1[(a_0 + a_1P_t^e + a_2X_t + U_{1t}) + S_{t-1} - CG_t] + b_2Z_t + U_{2t} \qquad \text{if } P_t \leq \overline{P}_t \qquad (4.4.8)$$

Now if there were no price supports, the market would have cleared in all time periods and CG_t, the amount of the crop acquired by the government under the price support program would have been zero.

Equations (4.4.7) and (4.4.8) would then reduce to

$$P_t = b_0 + b_1[(a_0 + a_1P_t^e + a_2X_t + U_{1t}) + S_{t-1}] + b_2Z_t + U_{2t} \qquad (4.4.9)$$

Taking expectations of both sides, we get

$$P_t^e = (b_0 + a_0b_1) + (1 - a_1b_1)^{-1}(b_1S_{t-1} + b_1a_2X_t^* + b_2Z_t^*) \qquad (4.4.10)$$

where X_t^* and Z_t^* represent the expectations of the supply and demand shifters at time t-1 when production decisions are made.

However, in case of price support program, the support prices are announced long before the planting season for the crop so as to permit the producer to plan and gear his farm operations to appropriate programs. Therefore, the price expected by the producers must equal or exceed the support price, i.e., $P_t^e \geq \overline{P}_t$ where $\overline{P}_t$ is the support price announced by the government. It should, however, be noted that this is true for any mechanism of expectation formation, i.e., the agents from their expectation and compare it with the announced support price and then make production plans on the basis of whichever is greater. Therefore, the expression for expectation that ultimately goes to influence the production decision is given by $\hat{P}_t^e$, where

$$\hat{P}_t^e = \mathrm{Max}(P_t^e, \overline{P}_t) \tag{4.4.11}$$

Now, in a market under price support program, the producers would form their expectations regarding whether the support price is going to be effective. If the support price is not effective, the producers receive the market price. But in view of equation (4.4.10), the market equilibrating price itself is a function of the price expectation P_t^e. Therefore, if $\pi_t = P_t(P_t > \overline{P}_t)$ i.e., the equilibrium price is higher than the support price, then

$$\begin{aligned}\pi_t = \Pr\{b_0 + b_1\{(a_0 + a_1 P_t^e + a_2 X_t + U_{1t}) + S_{t-1}\} \\ + b_2 Z_t + U_{2t} > \overline{P}_t\}\end{aligned} \tag{4.4.12}$$

But since P_t^e itself depends on π_t, we cannot have a closed form expression for P_t^e.

Let $P_{1t}^e = E(P_t | P_t > \overline{P}_t)$ i.e., P_{1t}^e is the rational expectation of P_t conditional on the assumption that the observed market price is greater than the price support $\overline{P}_t$, then P_t^e is given by

$$\begin{aligned}P_t^e &= E(P_t | I_{t-1}) \\ &= \pi_t P_{1t}^e + (1 - \pi_t)\overline{P}_t\end{aligned} \tag{4.4.13}$$

Now, to obtain P^e_{1t}, we have to take expectations of both sides of (4.4.9) conditional on $P_t > \overline{P}_t$. But, then the expectations of the residual does not vanish, since we are taking expectation over ψ_1 aone. Therefore, P^e_{1t} itself will involve P^e_t and as a result, one cannot obtain an explicit expression for P^e_t and even the implicit expression is quite complicated. However, we can still proceed with the estimation problem by assuming that

$$P_t = f(X^*_t, Z^*_t, S_{t-1}, \overline{P}_t) + V_t \text{ if } P_t > \overline{P}_t \qquad (4.4.14)$$

$$= \overline{P}_t \qquad \text{otherwise} \qquad (4.4.15)$$

where $f(X^*_t, Z^*_t, S_{t-1}, \overline{P}_t)$ is a non-linear function of X^*_t, Z^*_t, S_{t-1} and $\overline{P}_t$. In practice, we could approximate this by a qradratic function in these variables and estimate equations (4.4.14) and (4.4.15) by tobit mehtod. Let us denote the tobit estimates of the parameters of the approximating non-linear function by β and the standard error of V_t by σ_v. Then,

$$E(P_t | I_{t-1}, P_t > \overline{P}_t)$$

$$= \beta' X_t + E(V_t | I_{t-1}, V_t > \overline{P}_t - \beta' X_t)$$

$$= \beta' X_t + \sigma_v \frac{\phi_t}{1-\Phi_t} \qquad (4.4.16)$$

where ϕ_t and Φ_t represent the density function and the distribution function of a standard normal evaluated at $\frac{\overline{P}_t - \beta' X_t}{\sigma_v}$.

Therefore, using all the observations on P_t, we have

$$\begin{aligned} P^e_t = E(P_t|I_{t-1}) &= E(P_t|I_{t-1}, P_t > \bar{P}_t)\ P_r(P_t > \bar{P}_t) \\ &\quad + E(P_t|I_{t-1}, P_t < \bar{P}_t)\ P_r(P_t < \bar{P}_t) \\ &= (1 - \Phi_t)(\beta' X_t + \sigma_v \frac{\phi_t}{1-\Phi_t}) + \Phi_t \cdot \bar{P}_t \\ &= (1 - \Phi_t)\beta' X_t + \sigma_v\ \phi_t + \Phi_t \cdot \bar{P}_t \qquad (4.4.17) \end{aligned}$$

For P^e_t we can use equation (4.4.17) or alternatively, obtain the prediction of P^e_t from the tobit model directly, compare it with $\bar{P}_t$ in each period, and take the maximum of the two as an estimate of P^e_t. The predicted values are then substituted in the supply equation and the model is estimated in accordance with the procedure outlined above.

4.5 Futures Prices as Proxies for Expected Prices

As an alternative approach to estimating the supply response in a market in which agents' production decisions are based on expected price, it has been advocated in the literature to use the price of futures contracts for next year's crop (See, Gardner, 1976, Telser, 1967). There is a well-grounded theoretical literature that the price of a futures contract for next year's crop reflects the market estimate of next year's cash price. For example, studies by Dow (1940), Working (1942), and Tomek (1970) suggest that futures prices are unbiased point estimates of next year's cash prices. On the basis of his empirical analysis of wheat, corn and soybean markets in the U.S., Telser (1967) concludes that futures prices can be considered as an unbiased prediction of subsequent spot prices. Gardner (1976) studied the supply of soybeans and the acreage response in the cotton market in the U.S. and obtained reasonable results using futures prices that performed at least as well as Nerlovian lagged price, lagged dependent variable specification. Leuthold (1974, 1975), however, concluded to the contrary in his analysis of the price performance of the futures contracts in the live beef cattle market. He found that the livestock futures prices were poor estimates and systematically underestimated subsequent cash prices. In the light of such conflicting evidences, it should be of considerable interest to analyse the price performance of futures contracts in the corn market in the U.S. under price support programs.

There are, however, some conceptual problems associated with the use of futures prices. First, the "market's estimate" as obtained from futures price reflects the expectations of not only the producers of crop alone, but those of the non-farm speculators as well. Moreover, it is directly concerned with those producers only, who participate in futures transactions. Therefore, to use it to represent the expectations of all producers may not, strictly speaking, be justified. Gardner (1976) attempts to resolve this problem by appealing to the principle of rational expectations as developed by Muth (1961). He argued that if market expectations are formed rationally, then every agent in the market derives his expectation by taking into account the structure of the model of the market based on the information set available to them. Within this framework, there is no reason for the farmers' expectations, participating in futures contracts or otherwise, to differ from those of the non-farm speculators in the market. Otherwise, there is an incentive for the non-participants in the futures contracts to enter the futures market. Viewed in this light, the use of futures price can be justified as reflecting the market's estimate of expectation of prices. However, this explanation is consistent only if the agents in the market are assumed to form their expectations 'rationally' using 'all' the information available to them and on the basis of the structure of the model of the market in which they are operating. In such a framework, the expectations of the participants in the futures contracts are likely to coincide with those outside of futures market. However, if expectations in the market are formed in accordance with any alternative expectation schemes in which all the information available to the agents are not utilized in formation of expectations, then there may be problems in justifying the use of futures prices as reflecting the market's estimate of the next year's cash price.

There is also a possible scope for difference of opinion as to which futures price is most appropriate. This is because it is difficult to ascertain as to exactly at which point the production decisions are made. Moreover, even after the crop is planted, the planned output could be modified through actions in respect of fertilization, pest control and the like. However, the major decisions with regard to planting and acreage are made before planting and therefore futures contract prices immediately preceding the planting season can justifiably be taken as reflecting the producers' expectation of the next year's cash price. As corn is planted in the U.S. largely in the last week of April or in the first week in May and harvested in late October or early November, the futures prices prevailing in the last part of March was taken as an estimate of the price of corn as expected by the farmers in our empirical investigation. The end-of-March prices were used since it was felt that the corn-growers make their production decisions around that time leaving them about a month's time to act upon the decisions taken for making necessary arrangements in accordance with their plans. The futures prices used in our analysis are therefore the March 15 and March 30 prices of futures contract on the Chicago Board of Trade. These two prices were then averaged in order to smooth out single-day random price movements.

4.6 Conclusions

In this chapter we have discussed the nature of the corn price support program in the U.S. and the different formulations of the disequilibrium model that are appropriate for the analysis of the price support program. We have also discussed alternative schemes of expectation formation that can be incorporated into the model. The model for controlled prices presented in the previous chapter has been suitably modified to incorporate the special institutional features of the corn price support program. The empirical results are presented in the next chapter.

CHAPTER 5
A STUDY OF THE U.S. CORN MARKET UNDER PRICE SUPPORTS: EMPIRICAL RESULTS

5.1 Sources of the Data

The data used in our empirical investigation range from 1949 to 1981, and therefore we have all 33 observations. During this period, the corn market in the U.S. was subjected to a number of legislations governing its price support program. The details of such legislation are beyond the scope of the present discussion. These details are outlined in the U.S.D.A. Handbook No. 345. These regulations, insofar as they are relevant in case of corn, emphasize that the government should try to maintain a balance between the supply and demand in the market and the participants cooperating with the government in this attempt by reallocating their acreage will be brought under a price support pgoram under which the participants can obtain 'nonrecourse' loans for their crop stored with the government serving as collateral. Further, the government shall announce a support price for corn every year well ahead of the planting season in order to enable the producers to take appropriate actions on its basis. Now, if subsequently the market equilibrating price falls below the support price announced by the government, the participants in the support program may simply deliver or forfeit their crop with the CCC, which administers the support program on behalf of the government, and would not have any further obligations. If however, the market price is above the government support, the producers can sell their crop in the open market to repay their loan and pocket the difference. Thus, the price-support program in the corn market in the U.S. serves as a kind of price insurance for its participants for their crop.

The data for a number of variables viz. total production, stocks in all positions, exports, and average price received by the farmer and the support prices announced by the government in different years were obtained from various issues of Agricultural Statistics published by the United States Department of Agriculture. The data on the quantity of corn put under price support pgoram and the data on average yield per harvest acre for corn, sorghum and soybean were also obtained from various issues of Agricultural Statistics. The data on total disposable income were obtained from various issues of Survey of Current Business published by the Bureau of Economic Analysis in the United States Department of Commerce. The data on futures prices of contracts on the Chicago Board of Trade were obtained from various issues of the Wall Street Journal. The data on the rate of participation amongst the farmers in the price support program in the U.S. corn market were obtained from various issues of the Feed Situation Report.

Before entering into more elaborate discussion on the data used in our analysis, it is only fair to point out some possible limitations of the data. There are a number of possible sources of error associated with a given data set obtained from sample surveys. These include (i) errors of a conceptual nature arising out of faulty planning of the survey or incomplete specification for correction and processing of the data that may lead to inclusion of some items in wrong classes; (ii) errors due to voluntary misrepresentation that may arise out of uncertainty or fear of government authorities or from desire to deliberately mislead interested parties; or (iii) errors due to inadvertent misrepresentation that may arise because the subjects in the sample have misunderstood a question. For example, for the data on yield used in our analysis, the farmers are asked to specify the 'condition' of a season as a percent of 'normal', which is considered higher than that of most seasons but may be exceeded in an unusually favorable year. The question sought to elicit this response was phrased in a long and unclear manner, and is a possible source of error due to inadvertent misrepresentation.

The data on average price received by the farmers are to be viewed carefully. There are different grades in quality of corn in a market and quite naturally, they fetch different price. Hoever, the respondents are asked to give a sample price. They are thus, in effect, asked to quote a weighted average of these different prices for different grades of corn, where the weights should depend on the quantities of the different grade sold. This, however, is a difficult mental exercise and therefore, is a possible source of error due to inadvertent misrepresentation. Also, basic information on prices is obtained primarily from (1) country merchants, (2) farm produce dealers at the local shipping point, (3) country mill and elevator operators, (4) managers of local processing concerns, (5) co-operative marketing organizations,

(6) country bankers and (7) well-informed farmers, etc. Since the price statistics are often considered among the most carefully guarded secrets in the business community, it is a possible source of error due to voluntary misrepresentation. However, since these statistics are collected by the government from sources like, country bankers and managers of local processing concerns etc., who are well informed but may not directly consider the price of the crop to be maintained as a secret, the magnitude of this error may not be significant.

The data in Table 1 give the participation rates among the growers in the price support pgoram in the corn market. The data reveal the participation in the support program was rather poor till 1970, and even thereafter a singificant proportion of the farmers opted out of the program because of restrictions on acreage allotment, marketing quotas, etc. Since 1971, the acreage restrictions have been removed and this was responsible for a significant increase in the rate of participation that went up from 48.50 percent to 58.30 percent within 1970-71. These data, to a large extent, would determine the effectiveness of a government support program, i.e., to what extent the observed market price would fall below the support level. In the wheat or the cotton market in the U.S., for example, the rate of participation amongst the farmers is much higher and therefore, the observed market price in these markets does not fall significantly below the price support even when the market equilibrating price falls below the support price announced by the government.

Table 1

Data on Production, Stocks, Quantity Put Under Price Support,
Disposal Income and Total Exports in the U.S. Corn Market Over 1949-1981

Crop Year	Total[a] Production (million bushels)	Total[a] Stocks (million bushels)	Quantity[a] Put Under Price Support (million bushels)	Total[b] Disposable Income (billion dollars)	Total[a] Exports (million bushels)	Participation[c] Rates in the Price Support Program (in percent)
1949	2946	844	387	186	112	--
1950	2764	740	54	204	117	--
1951	2627	487	26	225	82	--
1952	2981	769	417	235	145	--
1953	2882	920	471	251	104	--
1954	2708	1035	259	254	103	40.00
1955	2873	1165	421	274	120	51.00
1956	3075	1419	477	290	184	44.00
1957	3045	1469	369	308	200	39.00
1958	3356	1524	381	316	230	--
1959	3825	1787	529	337	230	--
1960	3907	2016	637	350	292	--
1961	3598	1653	659	363	435	40.00
1962	3606	1365	591	384	416	38.30
1963	4019	1537	395	403	500	37.30
1964	3484	1147	216	440	570	38.90

Table 1 - continued

Crop Year	Total[a] Production (million bushels)	Total[a] Stocks (million bushels)	Quantity[a] Put Under Price Support (million bushels)	Total[b] Disposable Income (billion dollars)	Total[a] Exports (million bushels)	Participation[c] Rates in the Price Support Program (in percent)
1965	4103	840	215	472	687	38.90
1966	4168	823	264	509	487	44.20
1967	4860	1162	497	545	633	43.20
1968	4450	1113	404	590	536	46.50
1969	4687	1005	398	628	612	49.50
1970	4152	667	324	690	517	48.50
1971	5646	1126	955	744	796	58.30
1972	5580	709	420	795	1258	57.60
1973	5671	483	262	900	1243	62.40
1974	4701	359	77	990	1149	--
1975	5841	400	148	1100	1711	--
1976	6289	886	276	1180	1684	--
1977	6505	1111	1159	1305	1948	--
1978	7268	1304	641	1466	2133	--
1979	7939	1645	557	1645	2433	--
1980	6645	1034	840	1824	2355	--
1981	8202	2286	2150	2029	1967	--

Source:
a. Agricultural Statistics, U.S. Department of Agriculture, Various Issues
b. Survey of Current Business, U.S. Department of Commerce, Various Issues
c. Feed Situation, Various Issues

5.2 Discussion of the Data

A primary examination of the data would reveal that, over the period of our analysis, the production increased by nearly three times from 2946 million bushels to 8208 million bushels. The technological progress in agriculture can claim a large share of the credit for this dramatic increase in production. Interestingly enough, during this period the use of fertilizer has rapidly increased in the case of corn. Before the second world war, fertilizer used to be applied by corn growers only to a very limited extent and only in the southeastern part of the United States. But between 1942-1952 the use of fertilizer recorded a dramatic increase in Ohio, Indiana, Illinois, Iowa and Missouri, which are the major corn growing areas in the U.S. The use of hybrid corn, in fact, contributed the most towards this significant rise in production.

The data series on the average price received by the farmers is perhaps most interesting. In the 50's and 60's, the average price received by the farmer is found to be consistenly below the support price announced by the government except in 1950 and 1951. This may appear surprising in a market with price supports. One may wonder as to why the market price should be less than the support price at all. However, that the average market price is lower than the support level should not be taken to imply that all producers are receiving less than the support level. In fact, the participants in the support program have the opportunity to obtain the announced level of support any time during the period price support is available, regardless of whether the market price is below the support level or otherwise. Actually, under the price support program, the government's commitment is only to the eligible farmers participating in the support programs who will receive the support price through loans and purchases of their commodities. The government, however, has no commitment to maintain the market price at a specified level. Thus, the impact of the support program on the market is indirect rather than being direct. It should, however, be obvious that the impact of the support program can, in fact, hold the price at the support level, even when the equilibrating price is below the support price if only there is full participation of the producers in the support program. An examination of the data on the participation rates in the support program at the farm level in the corn market reveals that a significantly large proportion of farmers do not participate in the support program which explains, in part, why the market prices may be less than the price supports announced by the government.

Major reasons why prices sometimes fall below the support levels include (a) non-participation in the support programs due to lack of adequate storage facilities. This is because support in corn, administered through 'non recourse' commodity loan program (discussed in the previous chapter), requires that the participants must store the commodity either in farm storage facilities approved by ASC county office, or in a warehouse approved by CCC. Some producers, however, do not have adequate farm storage facilities, and therefore, to obtain support, they have to rely on warehouse storage facilities.

During periods of peak marketing and especially during periods of excess supply, it becomes quite difficult to obtain a suitable storage facility to come within the purview of the support program. Under such circumstances, the farmer may sell his commodity at the best available price, and such transactions contribute to lowering the observed market price below the support level; (b) if the market price is below but not very much different from the support level, the producer may choose to sell his crop at the best obtainable price. This is because the farmer who take out farm-storage loans is required to assume the costs of storage on the farm. Therefore, if the amount that he can receive by selling his crop at the best available market price is greater than the amount of loan repayment plus costs of storage, he would prefer to sell his crop at the market price and would not participate in the support program; (c) the market price may also fall below the support levels because some crops may not be of good storing quality and therefore the public storage facilities may refuse to accept their storage. Such crop may deteriorate unless they are sold in the market without delay and these transactions would tend to depress the market price. Further, the price support is also limited to merchandisable grades alone, and therefore there is always a part of the crop that is ineligible for being covered under price support program; (d) there are many small farmers whose production is not large enough to encourage them to participate in price support program. The cumulative effect of the total transactions of all such producers may significantly affect the level of market prices;

(e) some producers may choose not to participate in the price support program as such participation imposes restrictions on their acreage allotment, marketing quotas, etc., which they may not be agreeable to follow; (f) under the price support program, only an 'eligible' farmer, i.e., any person or other legal entity producing the crop being support as an owner, landlord, tenant or sharecropper may receive price support. However, in the course of transactions over a full marketing year, a considerable amount of the crop may be owned by non-producers, who would be ineligible to come within the purview of the price support program. They may sometimes choose to sell their stocks in a weak or falling market to minimize their losses. Taken collectively, such transactions would tend to depress the market price further below the support level; (g) although there are restrictions on sales of storable CCC-owned crop in the domestic market, such restrictions do not apply to the deteriorating stocks with the CCC. From time to time, CCC, therefore, releases stocks that have deteriorated or are in danger of deterioration and such actions would only tend to depress the observed level of market prices.

In the light of the disucssion above, we are in a position to appreciate why the observed market prices in 1950's and 1960's were significantly below the announced support level, as reflected in the data in table 2. However, it it is assumed that total demand in the market still comprises of the demand for all sources including the amount the government procures in price support programs in a period when the equilibrium price

Table 2

Data on Average Price, Support Price, and Futures Price in the U.S. Corn Market Over 1949-1981

Crop Year	Average Price as[a] Received by the Farmers (per Bushel)	National Average[a] Support Price (per Bushel)	Futures Price[b] as on 15th March (per Bushel)	Futures Price[b] as on 30th March (per Bushel)
1949	124	140	115	115
1950	152	147	119	125
1951	166	157	165	170
1952	152	160	176	176
1953	148	160	159	156
1954	143	162	149	146
1955	135	158	139	139
1956	129	150	135	140
1957	111	140	131	131
1958	112	136	115	116
1959	105	112	113	114
1960	100	106	110	111
1961	110	120	117	113
1962	112	120	118	119
1963	111	125	115	114
1964	117	125	123	119

Table 2 - continued

Crop Year	Average Price as Received by the Farmers (per Bushel)[a]	National Average Support Price (per Bushel)[a]	Futures Price as on 15th March (per Bushel)[b]	Futures Price as on 30th March (per Bushel)[b]
1965	116	125	121	121
1966	124	130	118	120
1967	103	135	138	141
1968	108	135	128	128
1969	116	135	113	114
1970	133	135	117	117
1971	108	135	153	143
1972	157	141	128	128
1973	255	164	146	147
1974	303	138	272	258
1975	303	138	255	274
1976	215	157	263	263
1977	202	200	269	264
1978	225	210	250	255
1979	252	220	256	264
1980	311	235	305	293
1981	250	240	370	379

Source: a. Agricultural Statistics, U.S. Department of Agriculture, Various Issues.
b. Wall Street Journal, Various Issues.

is below the support level, one essentially ignores the extent of government intervention in the market. It should be noted that when the price support is effective, the demand curve of the government becomes horizontal with respect to the participants in the support program. Therefore, to aggregate it with demand from other uses of corn may seriously bias the results. The extent of bias would be directly proportional to the extent of government intervention, which in turn, depends on the rate of participation of the farmers in the support program. The variable CG_t, to measure the government intervention under support program in the corn market when price supports are effective, is constructed from the data on the amount of corn acquired by the government through commodity loans and purchases. The data were obtained from various issues of Agricultural Statistics published by the U.S. Department of Agriculture. It should be recalled that the loans given to the participants in the support program are 'nonrecourse' in the sense that producers are not obligated to repay their loans or make good any decline in the market price of corn they have stored with CCC as collateral. Instead, a producer can deliver or forfeit his crop stored with CCC in settlement of his loan, if the market price falls below the support level. In fact, in such periods, the farmers would not find it advantageous to repay their loans and redeem the crop stored with CCC for sale in the market. During such periods, therefore, bulk of the loan stock is usually forfeited or delivered to CCC, that becomes a part of the CCC price-support inventory (Farm Commodity and Related Programs, U.S.D.A., Agricultural Handbook No. 345, p. 58). However, when the market price is above the support price offered by the government, it is always advantageous for the farmers to repay the loan and redeem their crop for sale in the open market. Thus, the participants in the support pgoram may and, in fact, do use the program as a kind of price insurance against any decline in the price of their produce below the support. We, therefore, construct our variable CG_t as follows. CG_t represents the amount procured by the government through loans and purchases during a period when the support is effective. On the other hand, when the market price is above the support level, CG_t is taken to be zero. This obtains support from the fact that during 1950's and 1960's, when the market price was significantly below the support level, CCC accumulated substantial inventories through forfeiture of stocks of the participants under the support program. But, when prices started looking up since 1972, CCC inventory registered sharp decline and was virtually eliminated by 1974 (see, Farm and Commodity Related Programs, U.S.D.A., Agricultural Handbook No. 345, p. 57). The data on futures prices were based on the 15th March and 30th March contracts on the Chicago Board of Trade. It is an important issue in using futures prices as reflecting the market participant's expectations, as to which futures contracts are most appropriate. This is because there is no preharvest date on which all the farmers can be said to have decided about their production plans. Further, production plans can be revised even after a

The data for our analysis are presented in Table 1, 2 and 3 respectively. To obtain the estimates of the price as expected by the farmers under the principle of rationality, the expected values of supply and demand shifters are required. These autoregressive and trend models are presented in Table 4. The high value of the R^2 for these models indicate that they do a respectable job for explaining the exogenous variables over the period of these models. These were then used to replace Y_t^*, DY_t^*, and XP_t^* in the system to obtain the expected price under rational expectation hypothesis with a tobit procedure as discussed in the previous chapter.

The expected price series were also obtained under other mechanisms of formation of expectation. Under extrapolative expectation hypothesis the expected price was generated by an AR(3) process which the value of R^2 was in high as 0.9713. The adaptive expectations were generated by Cagan's search procedure in which different values for the coefficient of expectation, $0 < \lambda < 1$ (where λ is the coefficient of expectation), were tried in constructing the expectation series P_t^e and then Q_t was regressed on P_t^e. The value of λ, for which R^2 turned out to be the maximum, was then chosen for our purpose. This value was 0.857 in our analysis of the corn.

The futures price expectations were obtained by averaging the March 15 and March 30 settlement prices for the December contract for corn on the Chicago Board of Trade. The rationale for choosing these contracts have already been discussed earlier.

Table 3

Data on Yield Per Acre in Corn, Sorghum, and Soybean in the U.S. Over 1948-1981

Crop Year	Corn (bu/Acre)	Soybean (bu/Acre)	Sorghum (cwt/Acre)
1949	38.2	22.3	22.5
1950	38.2	21.7	22.6
1951	36.9	20.8	19.1
1952	41.8	20.7	17.0
1953	40.7	18.2	18.4
1954	39.4	20.0	20.1
1955	42.0	20.1	18.8
1956	47.4	21.8	22.2
1957	48.3	23.2	28.8
1958	52.8	24.2	35.2
1959	53.1	23.5	36.1
1960	54.7	23.5	39.7
1961	62.4	25.1	43.7
1962	64.7	24.2	44.1
1963	67.9	24.4	43.9
1964	62.9	22.8	41.7
1965	73.8	24.5	51.6
1966	73.1	25.4	55.8
1967	80.1	24.5	50.4
1968	79.5	26.7	52.6
1969	85.9	27.4	54.3
1970	72.4	26.7	50.4
1971	88.1	27.5	53.8
1972	97.0	27.8	60.7
1973	91.3	27.8	58.8
1974	71.9	23.7	45.1

Table 3 - Continued

Crop Year	Corn (bu/Acre)	Soybean (bu/Acre)	Sorghum (cwt/Acre)
1975	86.4	28.9	49.0
1976	88.0	26.1	49.1
1977	90.8	30.6	56.6
1978	101.0	29.4	54.5
1979	110.0	32.1	62.7
1980	91.0	26.4	46.3
1981	110.0	30.1	64.1

Source: Agricultural Statistics, U.S. Department of Agriculture, Various Issues.

Table 4

Expectational Models for Exogenous Variables in the U.S. Corn Market

Total Disposable Income

$DY_t = -20.721 + 1.124\ DY_{t-1}$ $R^2 = 0.9994$

Export of Corn

$XP_t = 59.01 + 0.999\ XP_{t-1}$ $R^2 = 0.9404$

Yield of Corn

$\hat{YLP}_t = -75.522 + 2.226\ YEAR$ $R^2 = 0.9573$

crop has been planted through actions in respect to fertilizers, pesticides, etc. One is, therefore, naturally confronted with the equestion as to which futures prices should be considered as reflective of the market's expectation. Since corn is grown in the U.S. during late April or early May, and harvested during late October or early November, it was felt that the prices of futures contracts during the last fortnight in March, would be most appropriate. This would allow to the corn growers some time to make necessary arrangements on the basis of their plans. The futures prices of the 15th March and 30th March settlement prices of the December contract were, therefore, obtained and averaged to smooth out single day random price movements in order to be used as an estimate of the producers' expectations of the next year's cash price.

5.3 Empirical Results

The empirical results in our analysis of the corn market in the U.S. under price support program have been obtained under six alternative mechanisms of expectation discussed earlier viz. (l) cobweb or 'naive' (ii) extrapolative (iii) adaptive (iv) rational (v) futures prices and (vi) a perfect foresight approach in which information about the exogenous variables are assumed to be known to the producers at the time when planting decisions are to be made and one can, therefore, replace P_t^e by P_t in the supply equation. We essentially attempt to examine how expectations are formed in the U.S. corn market and the extent to which the price support program of the government affects the supply and demand side of the market.

In all the alternative schemes of expectations, it was assumed that the farmers expect to receive the support price $\overline{P}_t$, in view of the support program, whenever the expected price turns out to be less than the price support announced by the government. This is reasonable because over the years, the participation in the support program has gone up in the U.S. corn market, especially since 1971 when the acreage restraints, as required under the price support programs earlier, were removed. Theoretically speaking, however, since the market comprises two categories of producers viz. participants and non-participants in the support program, we tried to capture the differences in their supply response in the following manner. The supply of corn essentially has two components (i) supply by the participants and (ii) supply by the non-participants in the support program. The supply by the participants is determined by the price they expect to prevail in the market when the crop is harvested or $\overline{P}_t$, whichever is greater. But the supply by the non-participants is determined by the expected price alone, since they would not possibly expect to receive the support price announced by the government. The aggregated supply curve should therefore, theoretically speaking, be of the following form.

$$Q_t = a_0 + a_1' P_t^e + a_1'' \operatorname{Max}(\overline{P}_t, P_t^e) + a_2 \hat{YLD}_t + U_{1t}$$

i.e.,

$$Q_t = a_0 + a_1 \operatorname{Max}(\overline{P}_t, P_t^e) + \tilde{a}_1 Z_t + a_2 \hat{YLD}_t + U_{1t} \qquad (5.3.1)$$

The futures price also turns out to be a good proxy for price expectations as held by the farmers in the U.S. corn market. Our results are in sharp contrast with the observations of Leuthold (1974, 1975) who found prices of futures contracts to be a poor proxy of subsequent cash prices in his study of the live beef cattle market. The usefulenss of futures prices as unbiased prediction of subsequent spot prices have, however, been emphasized in earlier studies by Telser (1967) in his analysis of wheat, corn and soybean markets in the U.S. and Gardner (1976) who examined the supply of soybeans and acreage response in the U.S. cotton market. Interestingly enough, all these studies pertain to the U.S. agriculture sectors which is relevant for our purpose. The agreement of our results with their findings therefore strengthen our conclusion that in the corn market in the U.S. under price support program, prices of futures contracts can be taken to be an unbiased prediction of subsequent cash prices.

It is interesting to note that 'naive' expectation mechanisms, such as 'cobweb' or 'extrapolative' models have almost as much explanatory power as the more complicated Nerlovian adaptive expectations framework. Similar evidence was found by Shonkwiler (1982) in his study of the supply side of the Florida escarole market in which the cobweb price expectation model performed better than rational or partially rational mechanism of formation of expectations. Lastly, the perfect foresight model, in which the agents are assumed to have complete information of the future values of the exogenous variables at the time when production decisions are taken, where $a_1 = (a_1' + a_1'')$; $\tilde{a}_1 = -a_1'$

and $Z_t = \overline{P}_t - P_t^e$ if $P_t^e < \overline{P}_t$

$= 0$ otherwise

We tried to estimate the market supply schedule of corn with the above formulation, but $\tilde{a}_1$ turned out to be insignificant. In fact, the t-ratio for $\tilde{a}_1$ was less than 1 which meant that the $\overline{R}^2$ would actually decrease if Z_t is included in the supply specification. We therefore dropped Z_t from (5.3.1). The results indicate that even the non-participants in the support program, at least implicitly, utilize some information regarding the price support announed by the government in the formation of their expectation as to what the market price was going to be. A supply specification such as (5.3.1) without Z_t, therefore, adequately describes the supply side of the U.S. corn market under price support program. The expected price series under alternative mechanisms of formation of expectation were then obtained and they have been presented in Table 5.

The maximum likelihood estimates of the structural parameters are presented in Table 6. The parameters estimates of both the supply and demand equations have the expected signs and adequate degree of explanatory power. In terms of the value of the log-likelihood function, the rational expectations model performs the best but this is to be expected since the expectations are "model consistent" and there are some constraints (arising from the structure of the model) that are imposed in the expression for expected prices.

Table 5

Price of Corn as Expected by the Farmers Under Alternative Expectations Mechanisms: 1949-1981

Crop Year	Cobweb	Extrapolative	Adaptive	Rational	Futures
1949	140	140.0	140.0	140.0	140.0
1950	147	147.0	147.0	160.787	147.0
1951	157	157.0	157.0	157.0	167.5
1952	166	160.0	160.0	160.0	176.0
1953	160	160.0	160.438	160.0	160.0
1954	162	162.0	162.0	162.0	162.0
1955	158	158.0	158.0	158.0	158.0
1956	150	150.0	150.0	150.0	150.0
1957	140	140.0	140.0	140.0	140.0
1958	136	136.0	136.0	136.0	136.0
1959	112	112.0	112.186	112.0	113.5
1960	106	106.0	106.836	106.0	110.5
1961	120	120.133	120.0	120.0	120.0
1962	120	121.963	120.0	120.0	120.0
1963	125	125.0	125.0	125.0	125.0
1964	125	125.0	125.0	125.0	125.0
1965	125	125.0	125.0	125.0	125.0
1966	130	130.0	130.0	130.0	130.0
1967	135	135.0	135.0	135.0	139.5
1968	135	135.0	135.0	135.0	135.0
1969	135	145.606	135.0	135.0	135.0
1970	135	136.592	135.0	135.0	135.0
1971	135	135.0	135.0	135.0	148.0
1972	141	141.0	141.0	161.069	141.0
1973	164	184.779	164.0	176.903	164.0
1974	255	269.884	240.001	238.853	265.0

Table 5 - continued

Crop Year	Cobweb	Extrapolative	Adaptive	Rational	Futures
1975	303	278.403	293.944	238.900	264.5
1976	303	268.737	301.666	157.185	263.0
1977	215	200.0	227.337	219.679	266.5
1978	210	215.023	212.388	226.348	252.5
1979	225	222.816	222.107	239.673	260.0
1980	252	240.499	247.617	256.052	299.0
1981	311	309.195	301.859	259.102	374.5

Table 6

Maximum Likelihood Estimates of the Structural Parameters of the U.S. Corn Market Under Alternative Price Expectation Mechanisms

	Cobweb Expectations		Extrapolative Expectations	
Parameter	Est. Value	Asy. Std. Error	Est. Value	Asy. Std. Error
a_0	-756.660	302.975	-767.285	287.950
a_1	7.484	1.623	8.212	1.584
a_2	57.622	4.402	55.871	4.301
b_0	323.957	36.220	329.295	37.234
b_1	- 0.067	0.011	- 0.069	0.011
b_2	0.140	0.035	0.143	0.036
b_3	0.125	0.025	0.127	0.026
Log Likelihood	-404.6130		-403.3921	

Table 6 - continued

Parameter	Adaptive Expectations		Futures Expectations	
	Est. Value	Asy. Std. Error	Est. Value	Asy. Std. Error
a_0	-795.610	316.479	-661.316	246.711
a_1	7.940	1.755	8.142	1.264
a_2	57.207	4.594	53.904	3.889
b_0	320.403	39.920	319.165	35.972
b_1	- 0.066	0.012	- 0.066	0.011
b_2	0.138	0.039	0.140	0.035
b_3	0.123	0.027	0.121	0.025
Log Likelihood	-404.9553		-400.1259	

Table 6 - continued

Parameter	Perfect Foresight		Rational Expectations	
	Est. Value	Asy. Std. Error	Est. Value	Asy. Std. Error
a_0	-634.772	342.373	-1470.71	274.116
a_1	9.655	1.960	13.082	1.726
a_2	51.568	5.431	56.663	3.279
b_0	311.742	35.925	304.399	35.085
b_1	- 0.064	0.011	- 0.061	0.010
b_2	0.144	0.034	0.134	0.034
b_3	0.116	0.024	0.115	0.024
Log Likelihood	-660.5411		-398.1458	

gave results that were not significantly different from these obtained from the other models except for the coefficient a_1. Since this model is structurally the same as a model where P_t is substituted for P_t^e, this results shows that we can as well use P_t as a proxy for P_t^e. Note, however, that the log likelihood for this model is not comparable to that from the other models since P_t is endogenous in the supply function.

The elasticities as derived from the structural model naturally vary across the models. The range of variation is, however, not much compared to the wide spectrum of expectation mechanisms considered in our analysis. The estimates of the elasticities have been computed at the mean of the data and presented in Table 7. A significant difference across the models however, occurs when we compute the elasticity of supply relative to the price expectations held by the producers under alternative mechanisms of expectation. The results exhibit a strong relationship between the derived price expectations and supply. The response is mot pronounced for the model with rational expectations, viz. the elasticity of supply at the mean of the data is 0.441. The response turns out to be the poorest when the farmers are assumed to form their expectations in a naive cobweb manner. On the demand side, the price elasticity of demand for corn as implied by the model with rational expectations is -0.496. The demand elasticity as implied by the other models are close but less than the value implied under the reational expectation hypothesis. These results are much different from the estimated elasticity of

Table 7

Estimated Elasticities for the U.S. Corn Market Computed at the Mean of the Data

Elasticities	Cobweb	Extrapolative	Adaptive	Perfect Foresight	Rational	Futures
Elasticity of Supply Relative to Expected Price	0.264 (0.057)	0.309 (0.060)	0.294 (0.065)	0.345 (0.070)	0.441 (0.058)	0.315 (0.049)
Elasticity of Supply Relative to Yield/ Acre	0.855 (0.065)	0.859 (0.066)	0.879 (0.071)	0.793 (0.084)	0.871 (0.051)	0.829 (0.060)
Price Elasticity Demand	-0.452 (0.075)	-0.439 (0.070)	-0.459 (0.084)	-0.473 (0.082)	-0.496 (0.082)	-0.473 (0.077)
Income Elasticity of Price	0.586 (0.147)	0.603 (0.151)	0.578 (0.163)	0.599 (0.143)	0.561 (0.147)	0.586 (0.147)
Export Elasticity of Price	0.611 (0.123)	0.568 (0.127)	0.601 (0.132)	0.622 (0.118)	0.563 (0.167)	0.593 (0.123)

Figures in the parentheses represent the standard errors.

demand as obtained in previous studies on the corn market in the U.S. Foote, Klein and Clough (1952) reported the price elasticity of demand of corn in the U.S. to be -0.6. However, their results were derived without any consideration of the price support program in the corn market. In a recent study, Riboud and Rausser (1981) attempted to account for the government intervention in the corn market in the U.S. in their analysis of the demand side of the market. They employed a two-stage least squares and a modified tobit estimator for the purpose. The estimates of the price elasticity of demand of corn as obtained in their study are -0.52 and -0.55 respectively. The model developed by them is however appropriate only if all the corn producers participate in the support program which is far from reality. Especially in the corn market in U.S., there was a significant proportion of non-participants in the early years of the program when stringent restraints on the acreage had to be practiced in order to be eligible to participate in the support program. To a certain extent, this accounts for the observed market price to remain below the price support during the 1950's and 1960's as reflected in Table 2. This is taken care of in our approach by making appropriate corrections for the quantity procured by the government from the participants at the support price. No such correction is, however, made by Riboud and Rausser (1981) in their analsyis of the demand side of the corn market. Therefore, although their results are in closer agreement with our findings, we can reasonably expect that the estimates of demand elasticities as obtained in our study, are more accurate. Further, the income and exports both have positive impacts on the demand side. Both the elasticities are of the order of 0.6 for the models examined in our analysis. In fact, the export demand for corn has dramatically gone up since 1971 from a meager 4 percent in 1949. At present, it accounts for nearly 25 percent of the total production of corn in the U.S. Since as a general rule, CCC does not intervene in the export market when privately owned stocks are available for export (see USDA, p. 63), the demand for export has considerable influence in the determination of the price of corn in the U.S.

The reduced form parameter estimates have been presented in Table 8. The estimates are easy to compute owing to the familiar recursive structure of our model whenever the expected price is specified in terms of exogenous and predetermined variables. However, the model is not fully recursive since the contemporaneous correlation across the equations is not zero.

5.4 Summary and Conclusion

The results we have obtained from an estimation of the disequilibrium model taking into account the price support program and its effect on the formation of price expectations of the producers can be summarized as follows:

(1) The price support program does not cover all the farmers. But estimation of a model that takes this factor into account showed insignificant and minor changes in the parameters for the participants and non-participants (see equation (5.3.1). Thus,

Table 8

Derived Reduced Form Results for the Corn Market in the U.S. Under Alternative Mechanisms of Price Expectations

Dependent Variable	Model	Intercept	P^*_t	YLD	SL	CG	DY	XP
Q_t	Cobweb	- 756.66	7.484	57.622	0		0	0
	Adaptive	- 795.61	7.94	57.207	0		0	0
	Extrapolative	- 767.285	8.212	55.871	0		0	0
	Rational	-1470.71	13.082	56.663	0		0	0
	Futures	- 661.316	8.142	53.904	0		0	0
	Perfect Foresight	1467.99	---	31.873	-0.381922	0.381922	0.859326	0.692234
P_t	Cobweb	374.653	-0.501428	-3.86067	-0.067	0.067	0.14	0.125
	Adaptive	372.913	-0.52404	-3.77566	-0.66	0.066	0.138	0.123
	Extrapolative	382.238	-0.566628	-3.8551	-0.69	0.069	0.143	0.127
	Rational	394.112	-0.798002	-3.45644	-0.61	0.061	0.134	0.115
	Futures	362.812	-0.537372	-3.55766	-0.66	0.066	0.14	0.121
	Perfect Foresight	217.79	---	-2.03987	-0.039557	0.039557	0.0890032	0.071697

we conclude that even non-participants in the program, at least implicitly, utilize some information regarding the price support announced by the government in the formation of expectations as to what the market price is going to be.

(2) Our results indicate that futures prices are good proxies for expected prices. This is in sharp contrast with the observations of Leuthold (1974, 1975) but agrees with the results obtained in the studies of Telser (1967) and Gardner (1976).

(3) Naive schemes of expectation (such as cobweb and extrapolative) performed as well (in fact slightly better) than more complicated schemes of expectation, such as adaptive that has been so widely used in the estimation of supply response in agriculture (Askari and Cummings, 1977).

(4) Though the estimates of the parameters of the demand function did not change much between the different specifications of expectation formation, the elasticity of supply with respect to expected price was the highest with the rational expectations model and was the lowest with the cobweb type model. Thus, one might grossly underestimate the magnitude of supply response to changes in price if one used the naive or adaptive expectations model.

CHAPTER 6
SUMMARY AND CONCLUSION

The principal aim of this study is to discuss the estimation problems in disequilibrium models when we introduce expectational variables into the models. Specifically, we consider the incorporation of rational expectations of prices. The analysis depends on what the disequilibrium is due to. We consider two sources of disequilibrium:

(a) Partial adjustment of prices to their market equilibrating levels, and

(b) Limits in the movement of prices due to the existence of price controls or price supports.

In both these cases we introduce rational expectations of price when the quantity supplied depends on expected price. The term "rational" is used throughout in the same sense as that used in all the econometric work in this area. A better terminology is "mdoel consistent" but we have avoided this term to avoid any confusion. The rational expectations hypothesis implies that decision makers use all the information available to them at the time they are forming their expectations. Further, it is assumed that this information includes not only the past values of all the exogenous and endogenous variables but also the structure of the model that the econometrician is trying to estimate. Since this particular concept of 'rationality' may not be appealing to everyone, in the empirical work we have used alternative expectational schemes. Since these alternatives imply that the expected price can be treated as exogenous as far as the estimation of the particular model under consideration is concerned, there are no additional problems of estimation besides those already discussed in the literature. New problems arise only if we are considering rational expectations. Hence this is the case considered in detail.

In Chapter 2 we discuss the model with partial adjustment in considerable detail. We discuss the estimation of the model when we incorporate rational expectations, serial correlation in the residuals, spill-overs of past excess demand and excess supply into the current period, and inventories. All these are factors that have been ignored in the current econometric literature.

In Chapter 3 we discuss the model with controlled prices under rational expectations. We also discuss the estimation of a two-market model where excess demand from the market with price controls spills over into the market for a substitute good which is not subject to price controls and hence is in equilibrium. Since any further elaboration of these models depends on the details of the specification of the nature of the industry subject to price controls, we do not pursue this any further. Instead, we illustrate the methodology with reference to the price support program for corn in the U.S. This will illustrate how modifications can be made in the basic framework for the model with controlled prices and rational expectations presented in this chapter.

In Chapters 4 and 5 we discuss the U.S. price support program in corn and estimate a demand and supply model. When the price supports are effective, there is an excess supply but this is absorbed by governmental purchases. We assumed, as in earlier studies, that supply depends on expected price and that producers form their expectations on the basis of support prices that are announced in advance. The support price acts as a floor on what producers can expect. In addition to the "rational" expectations model we also tried other schemes of expectation formation: cobweb, extrapolative, and adaptive. We also estimated a "perfect foresight" model. The estimates of the parameters of the demand function were almost the same for all models. It is the price elasticity coefficient in the supply function that differed between them. The elasticity of supply with respect to expected price was the lowest for the cobweb model and the highest for the rational expectations model (twice as high as for the cobweb model). Earlier studies on supply response in agriculture did not explicitly model the existence of the price support program (they treated it in an ad-hoc fashion) and mostly used the adaptive expectations model. The present study corrects both these shortcomings by using a disequilibrium model and alternative formulations of expectations, including the rational expectations model.

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